

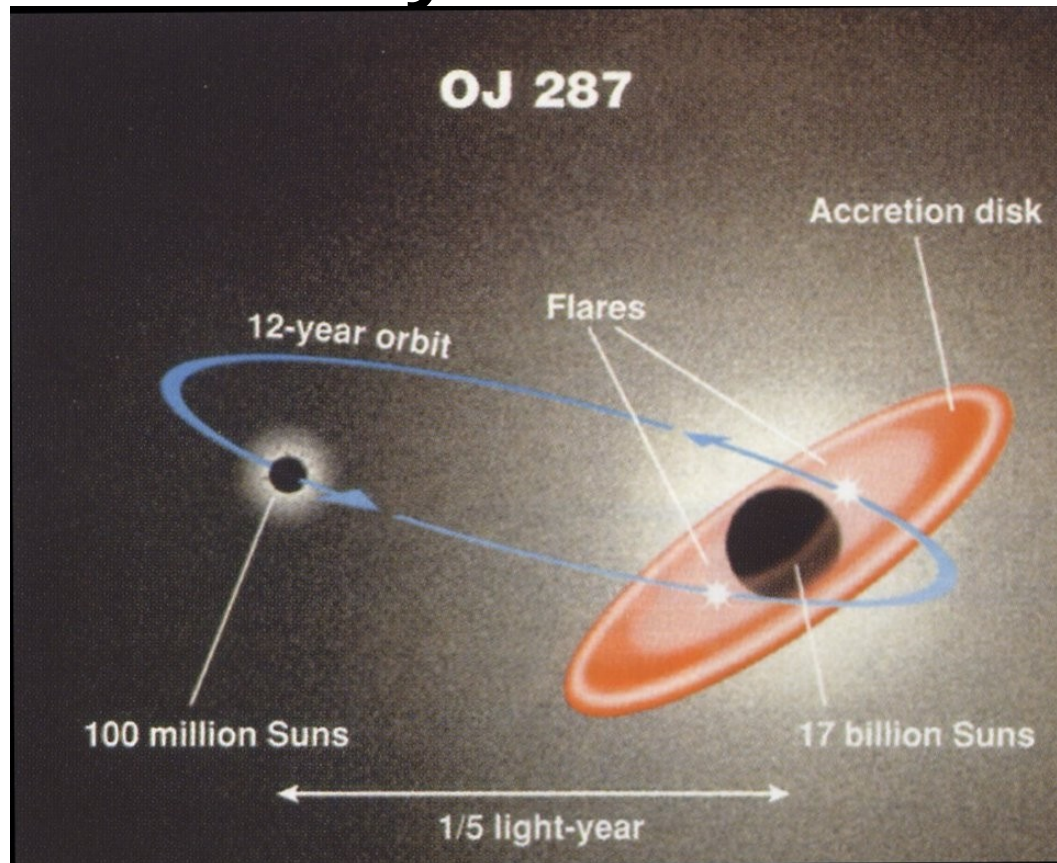
Multiwavelength campaign of OJ287 in 2005-2010

M. Valtonen, HIP

8th INTEGRAL/BART Workshop

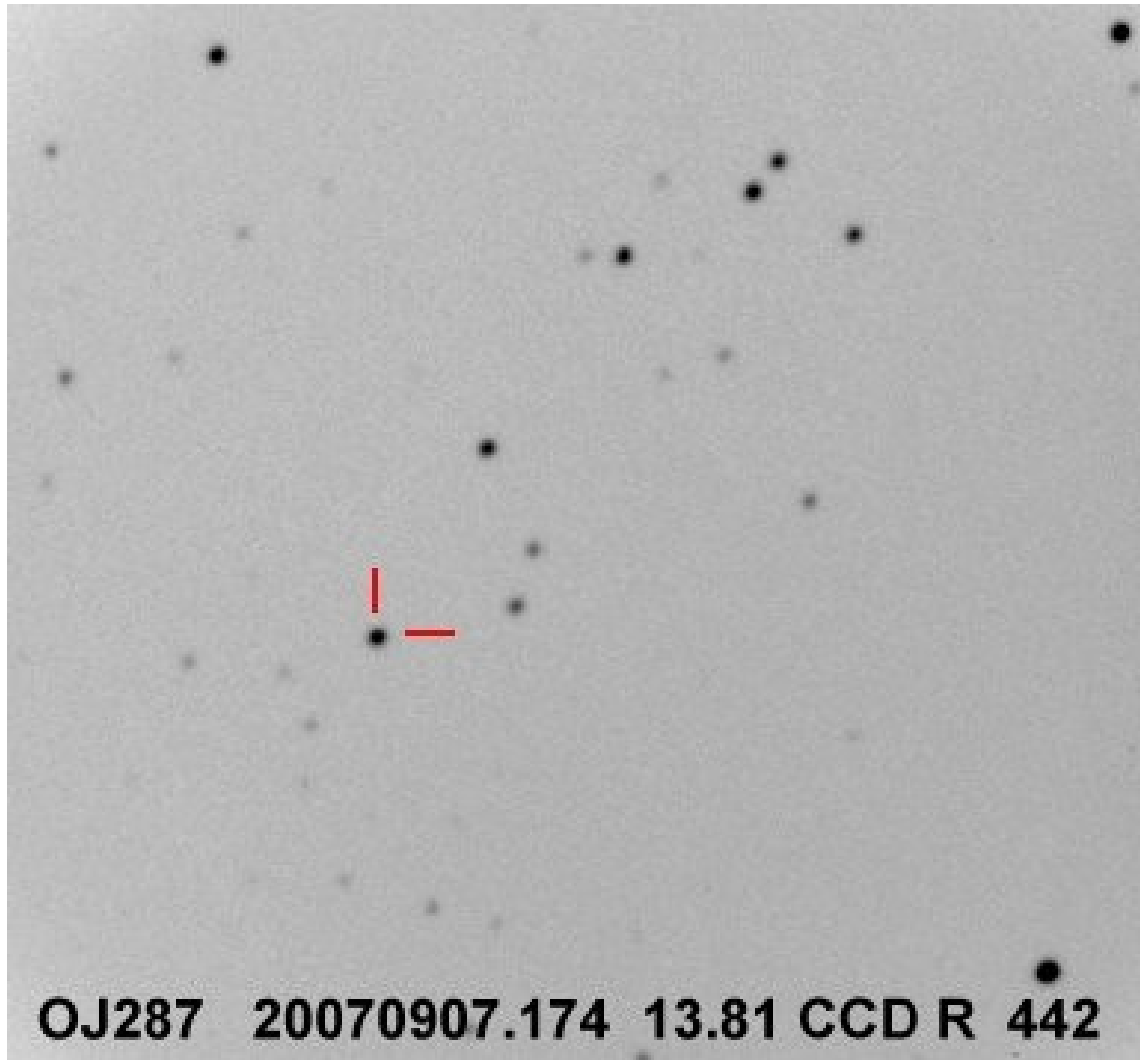
April 29, 2011

OJ287 binary black hole system?

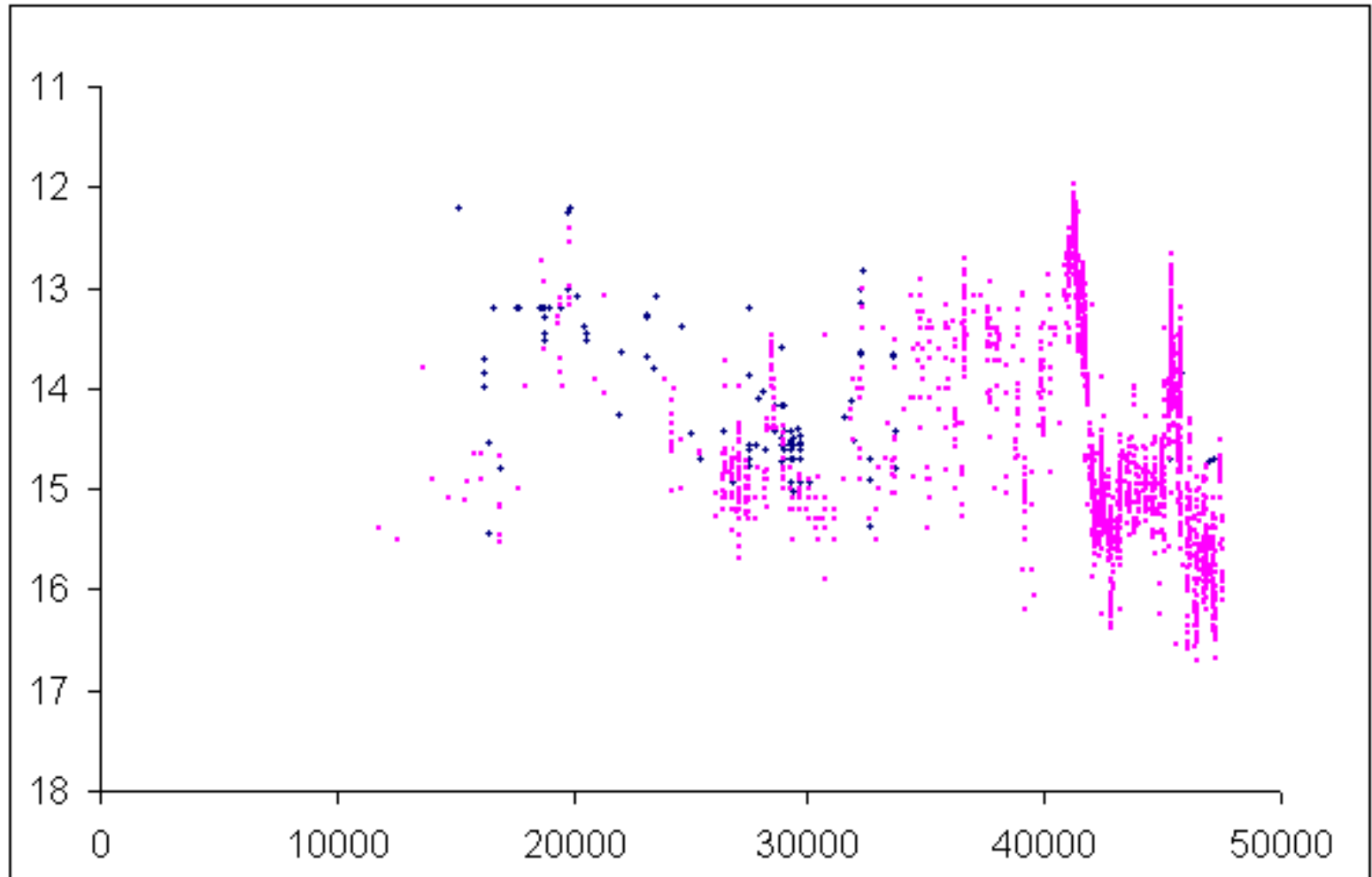


Sillanpää, Haarala, S., Valtonen, M. J., Sundelius, B. & Byrd, G. G., 1988, ApJ, 325, 628

Light curve



Optical light curve: 1891 -



1. Signature of binary black holes: precession

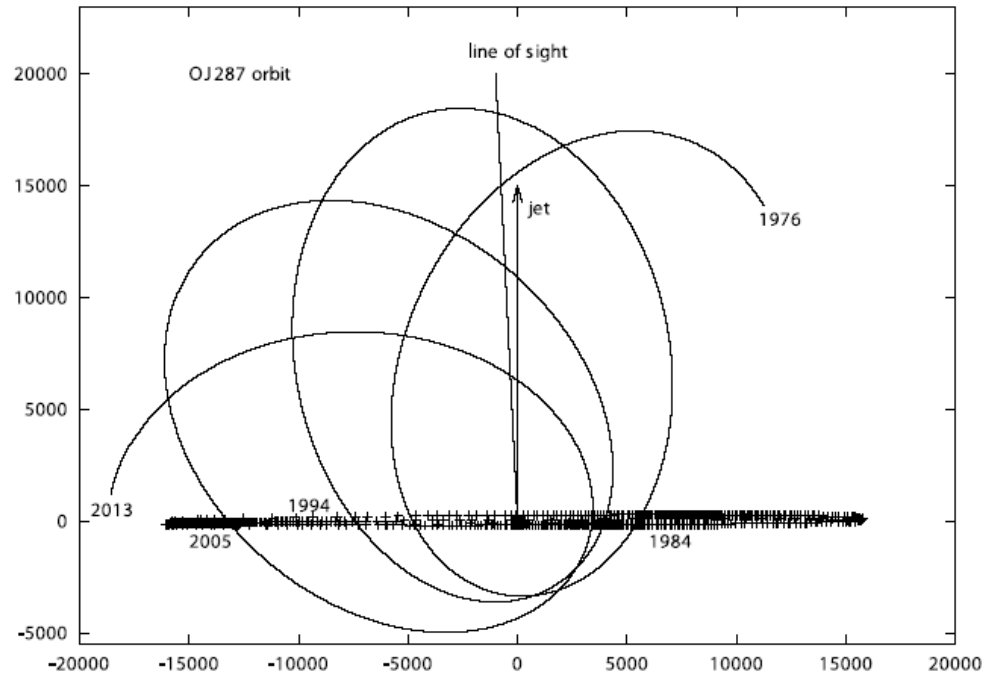
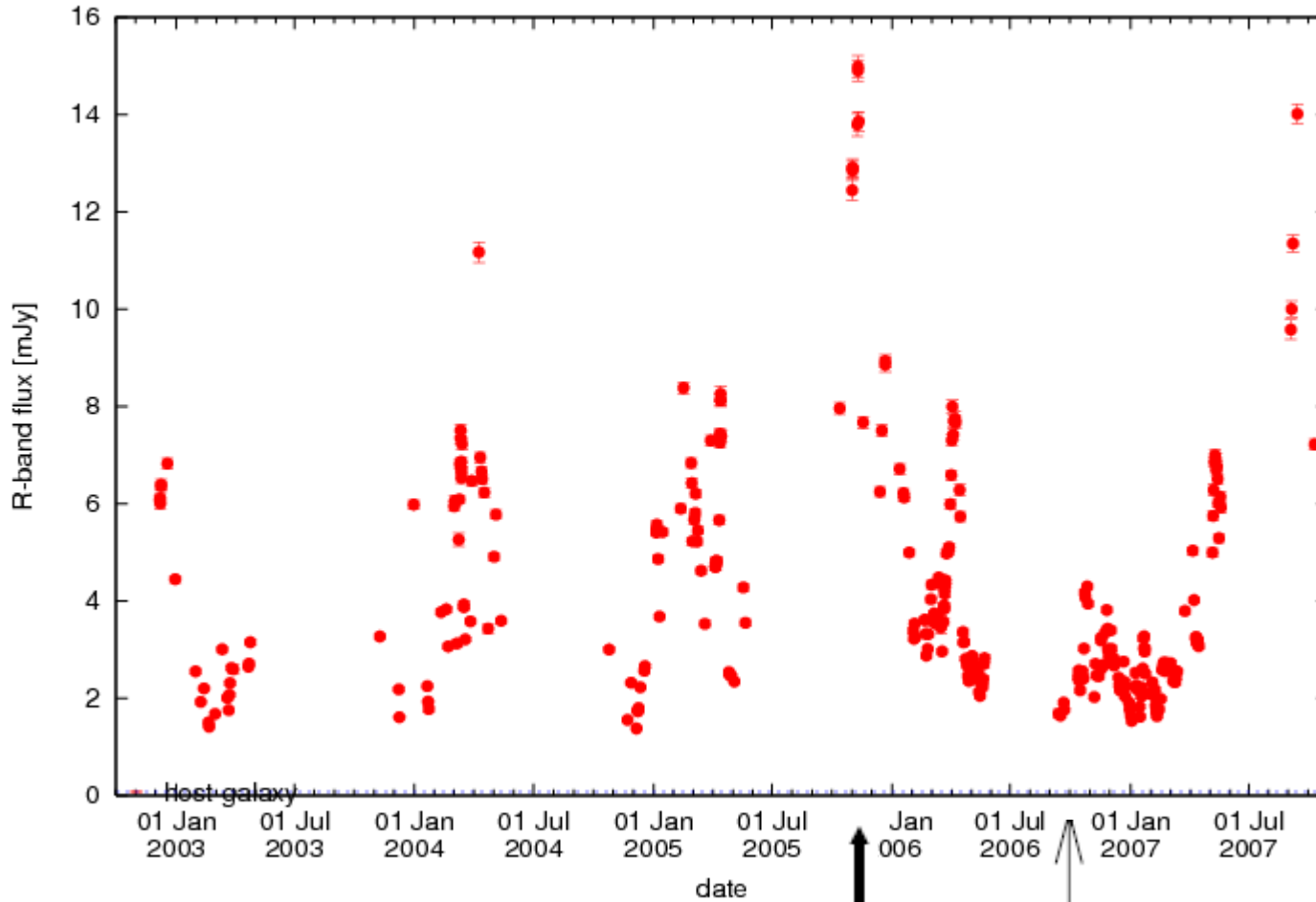


FIG. 1.—Precessing-binary model of OJ 287. The primary black hole is at coordinates $(0, 0)$. The secondary black hole traces out a precessing elliptical orbit counterclockwise around the primary, illustrated here from 1976 to 2013. The secondary crosses the accretion disk of the primary (*horizontal set of points*) at impact sites; the 1984, 1994, and 2005 impacts are labeled. The axis of the accretion disk is labeled “jet.” Not far from the jet is the line of sight to the observer. The binary orbit is at right angles to the accretion disk; Sundelius et al. (1997) have shown that this is not a restriction on the generality of the model, i.e., different relative inclinations would produce very similar results.

$$\Delta\phi = 39.1 \pm 0.1$$

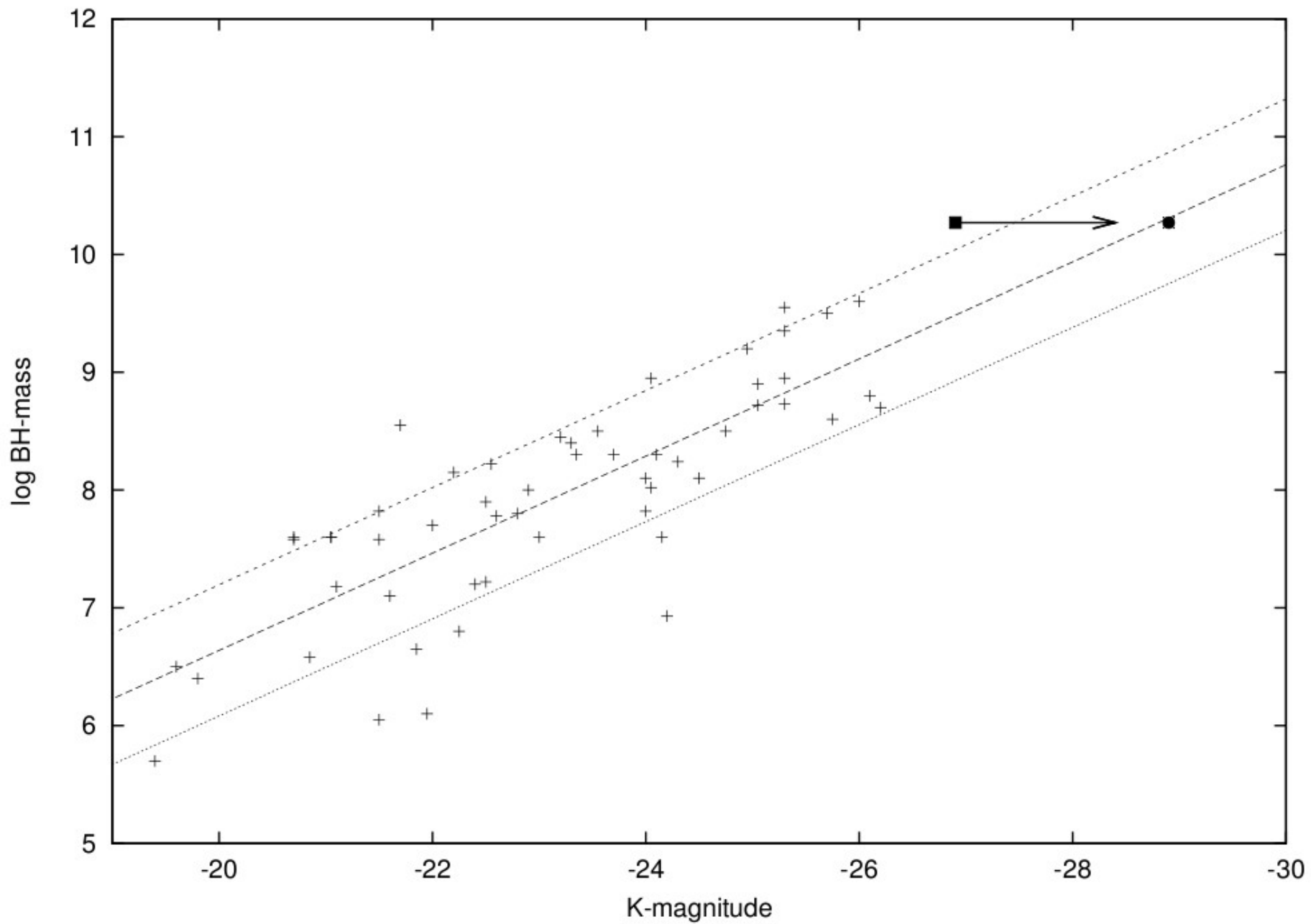
OJ_287



Sillanpää, A. et al. precession

no precession

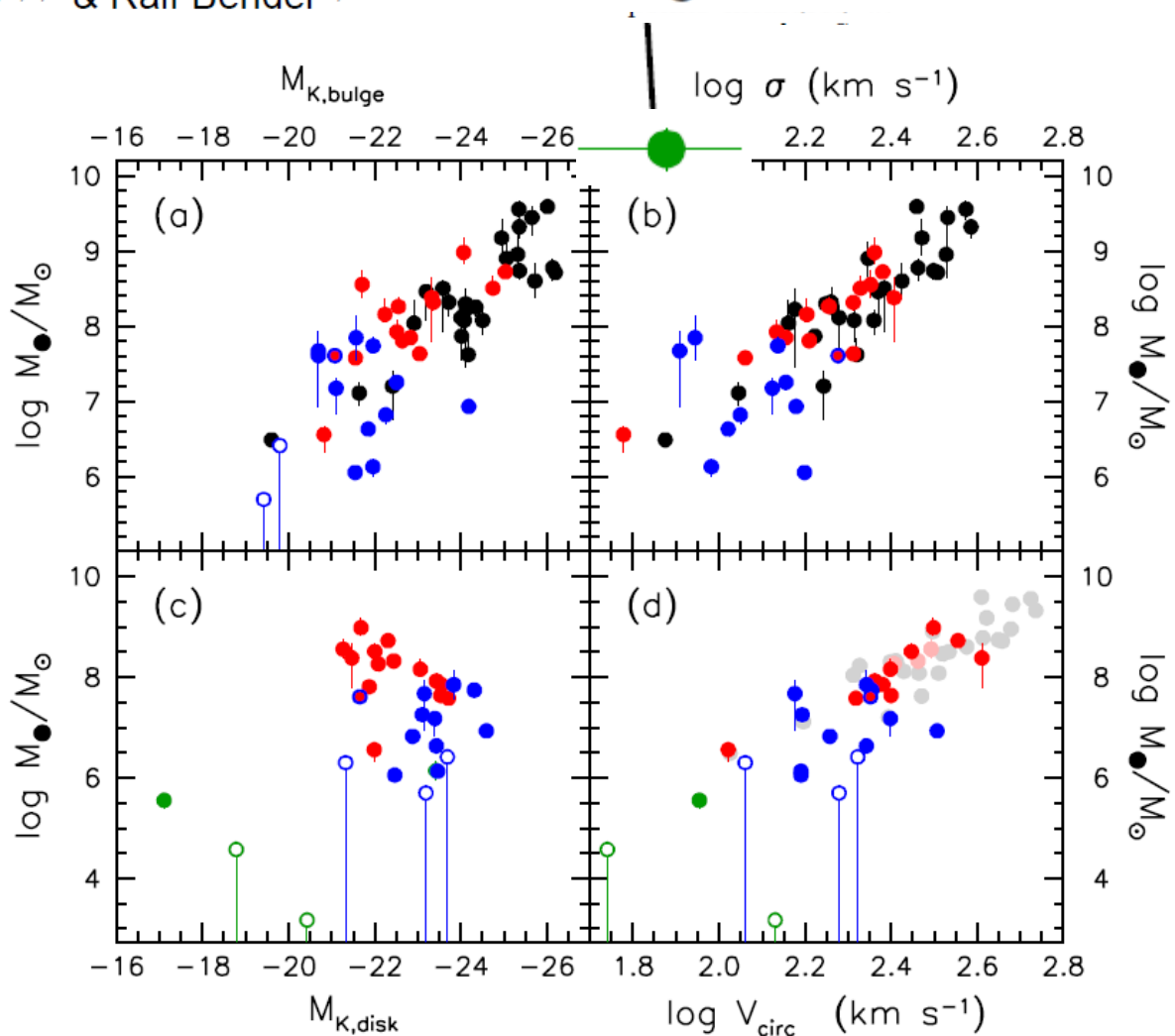
Igumenshev, I. V., & Abramowicz, M. A. 1999



Supermassive black holes do not correlate with dark matter halos of galaxies

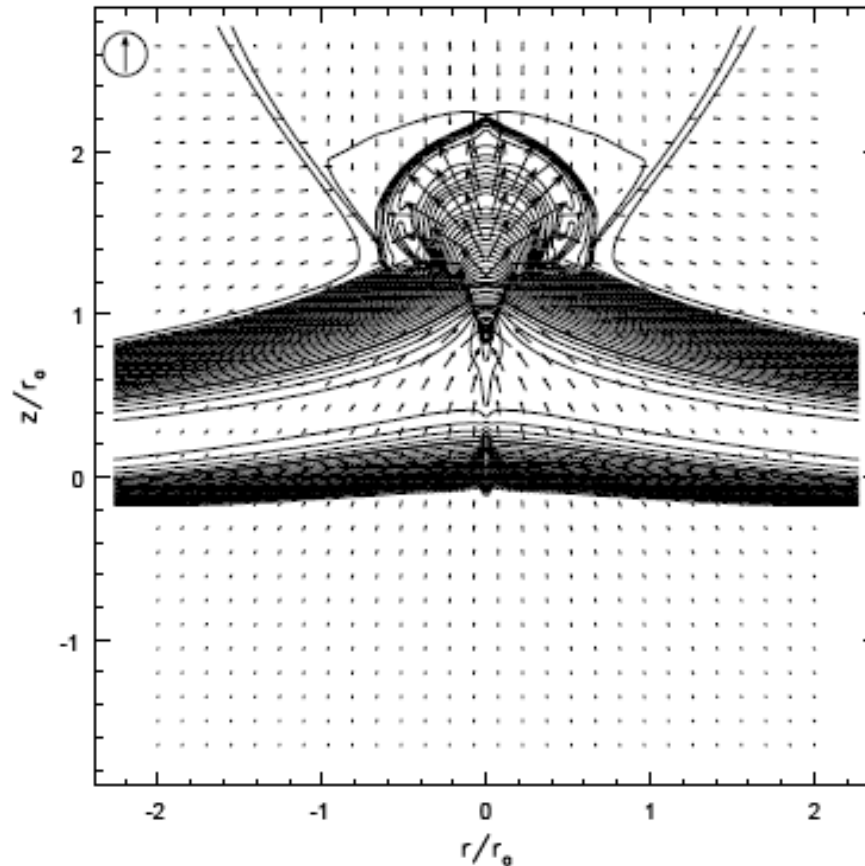
John Kormendy^{1,2,3} & Ralf Bender^{2,3}

OJ 287

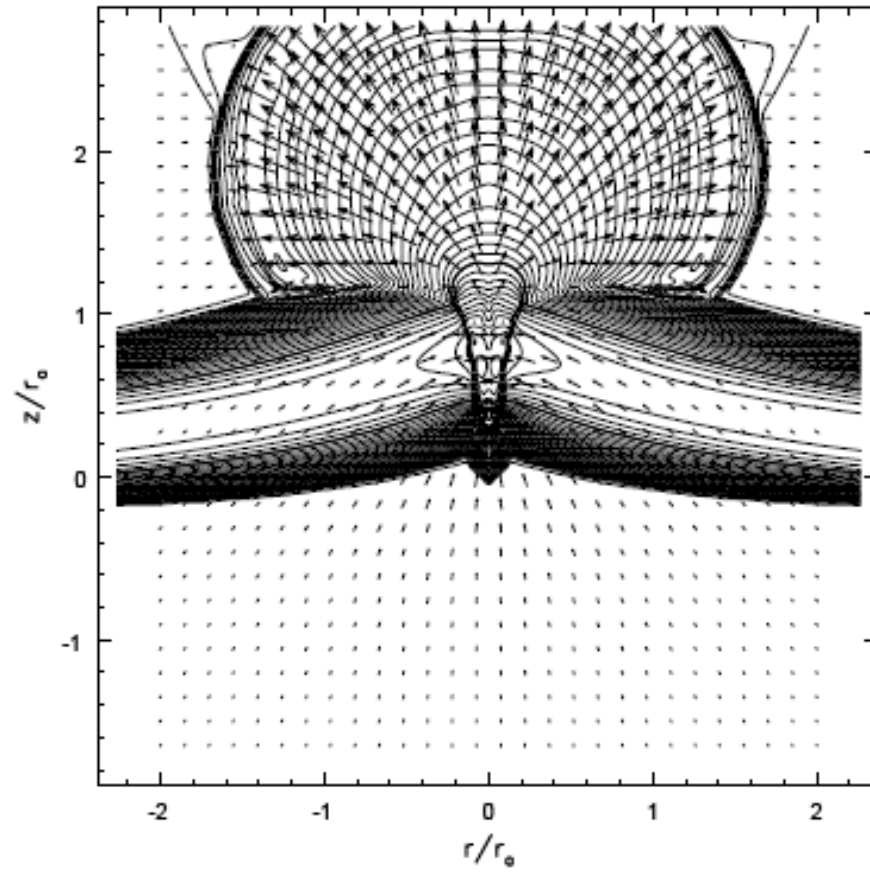


2. Black hole – Accretion disk collision

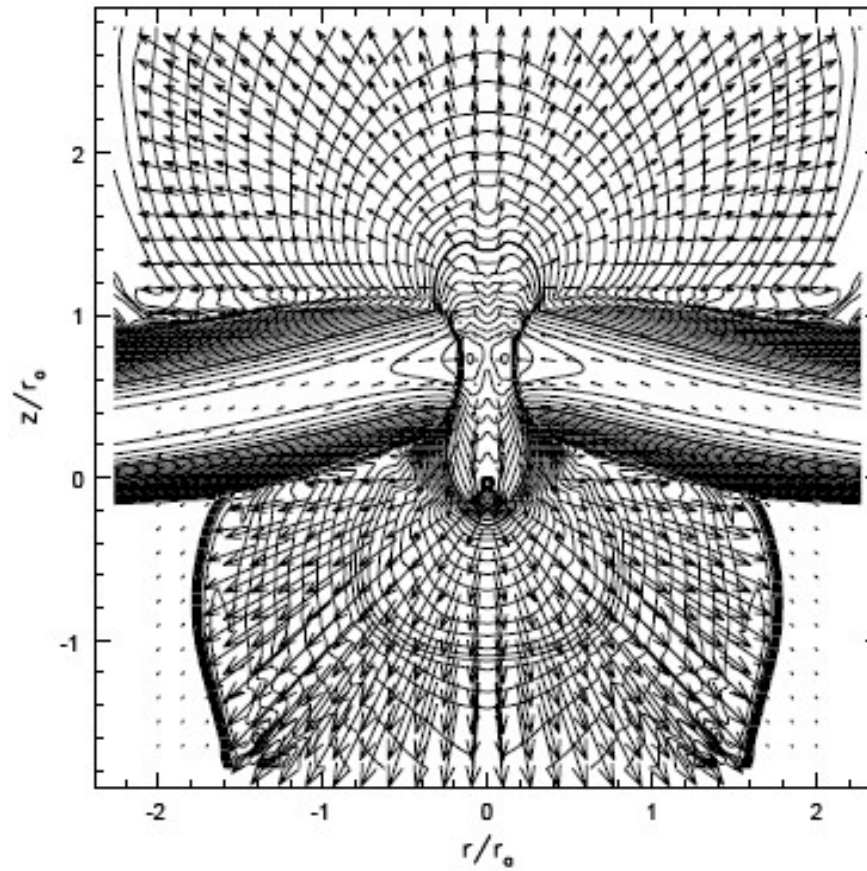
Ivanov et al. 1998

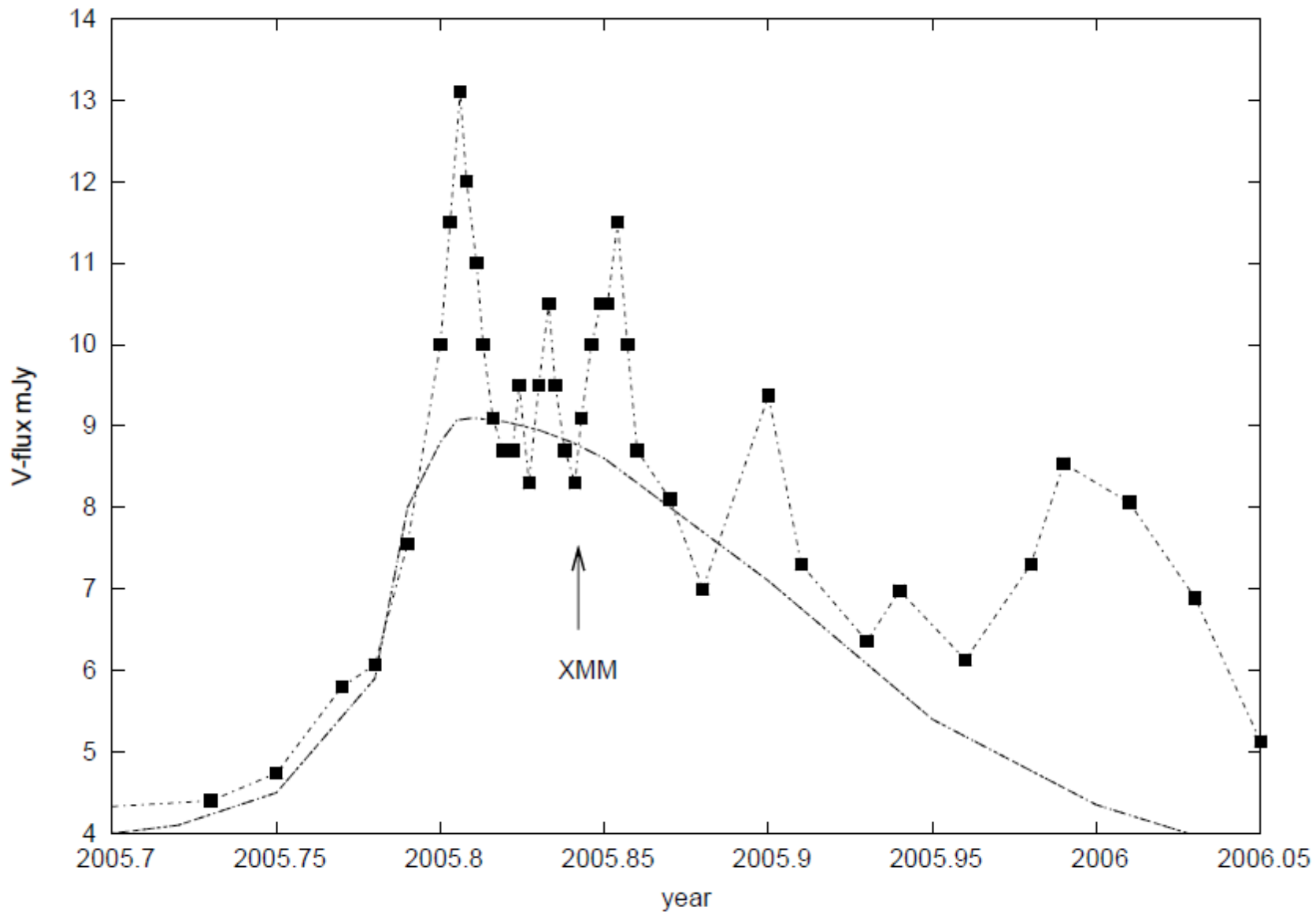


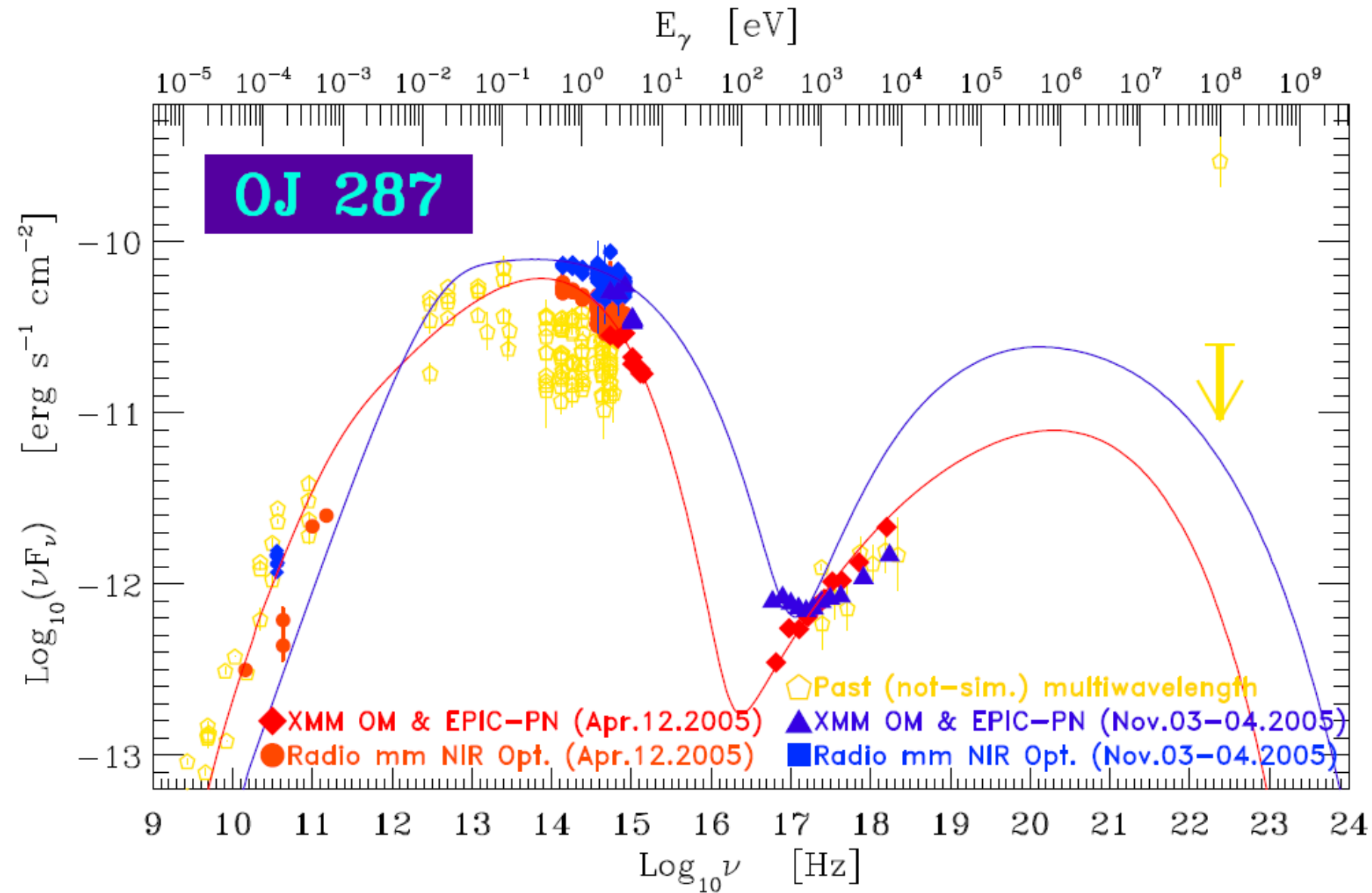
Collision 2

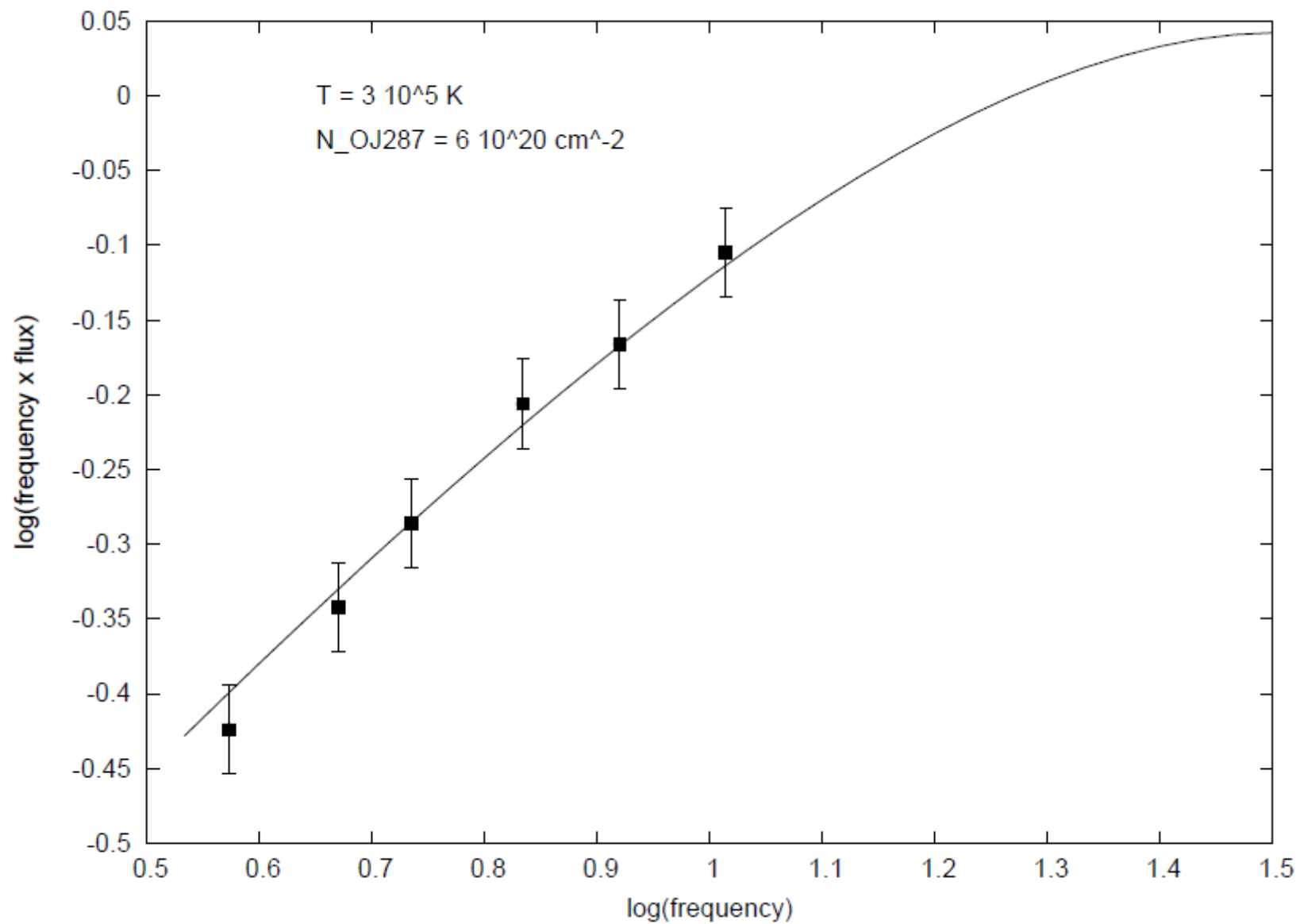


Collision 3





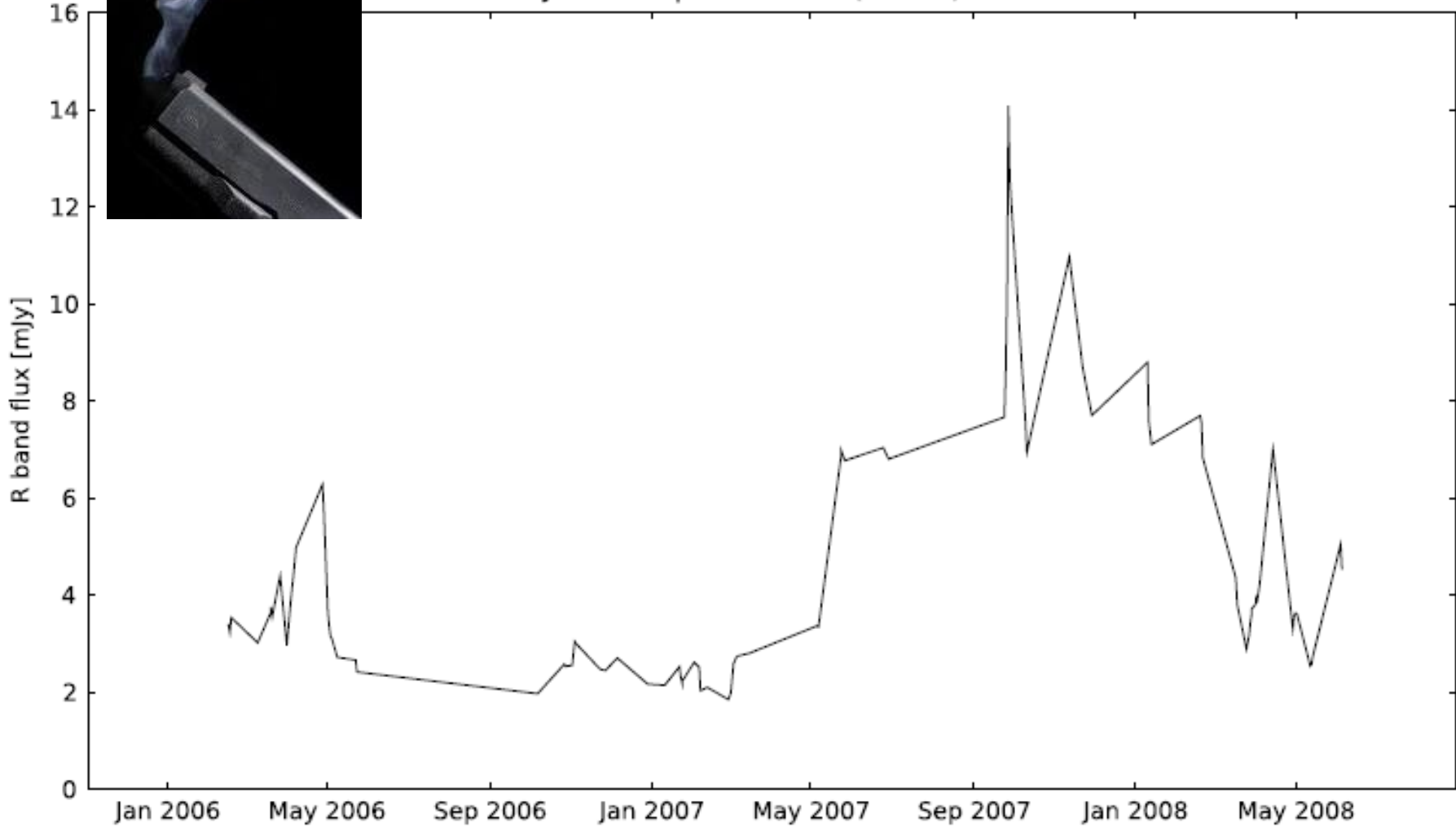




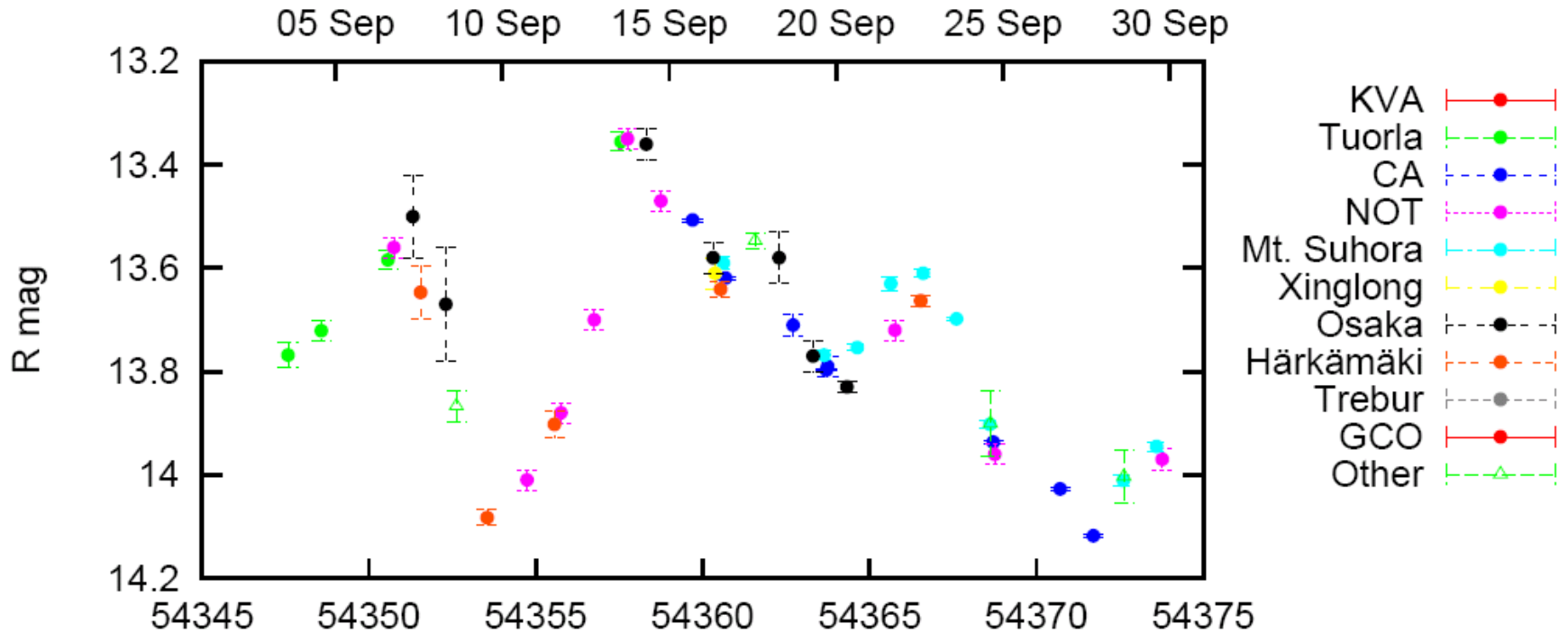
3. Prediction: 2nd outburst Sept 2007 unpolarized



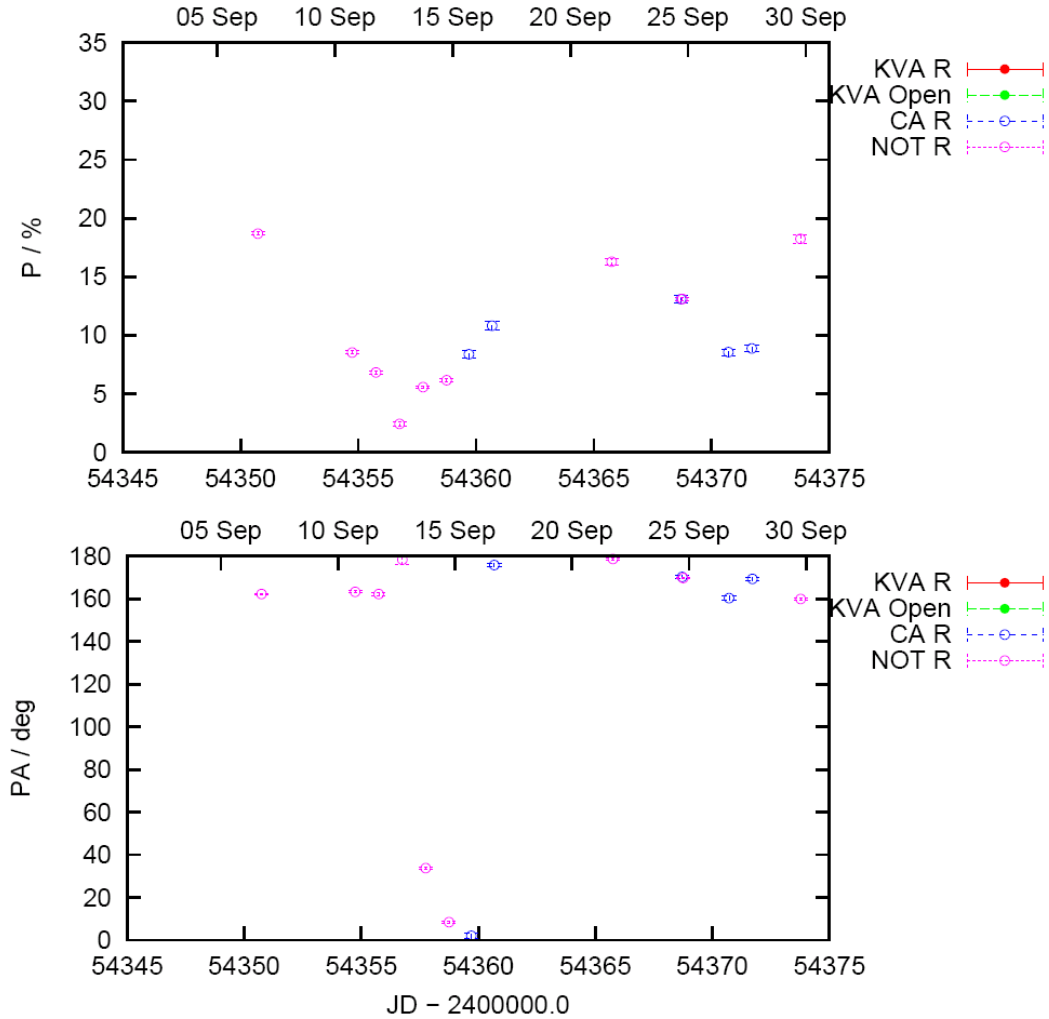
OJ287 low polarization (<10%) flux



OJ287 in September 2007



Polarization



A massive binary black-hole system in OJ 287 and a test of general relativity

M. J. Valtonen¹, H. J. Lehto¹, K. Nilsson¹, J. Heidt², L. O. Takalo¹, A. Sillanpää¹, C. Villforth¹, M. Kidger³, G. Poyner⁴, T. Pursimo⁵, S. Zola^{6,7}, J.-H. Wu⁸, X. Zhou⁸, K. Sadakane⁹, M. Drozd⁷, D. Koziel⁶, D. Marchev¹⁰, W. Ogloza⁷, C. Porowski⁶, M. Siwak⁶, G. Stachowski⁷, M. Winiarski⁶, V.-P. Hentunen¹¹, M. Nissinen¹¹, A. Liakos¹² & S. Dogru¹³

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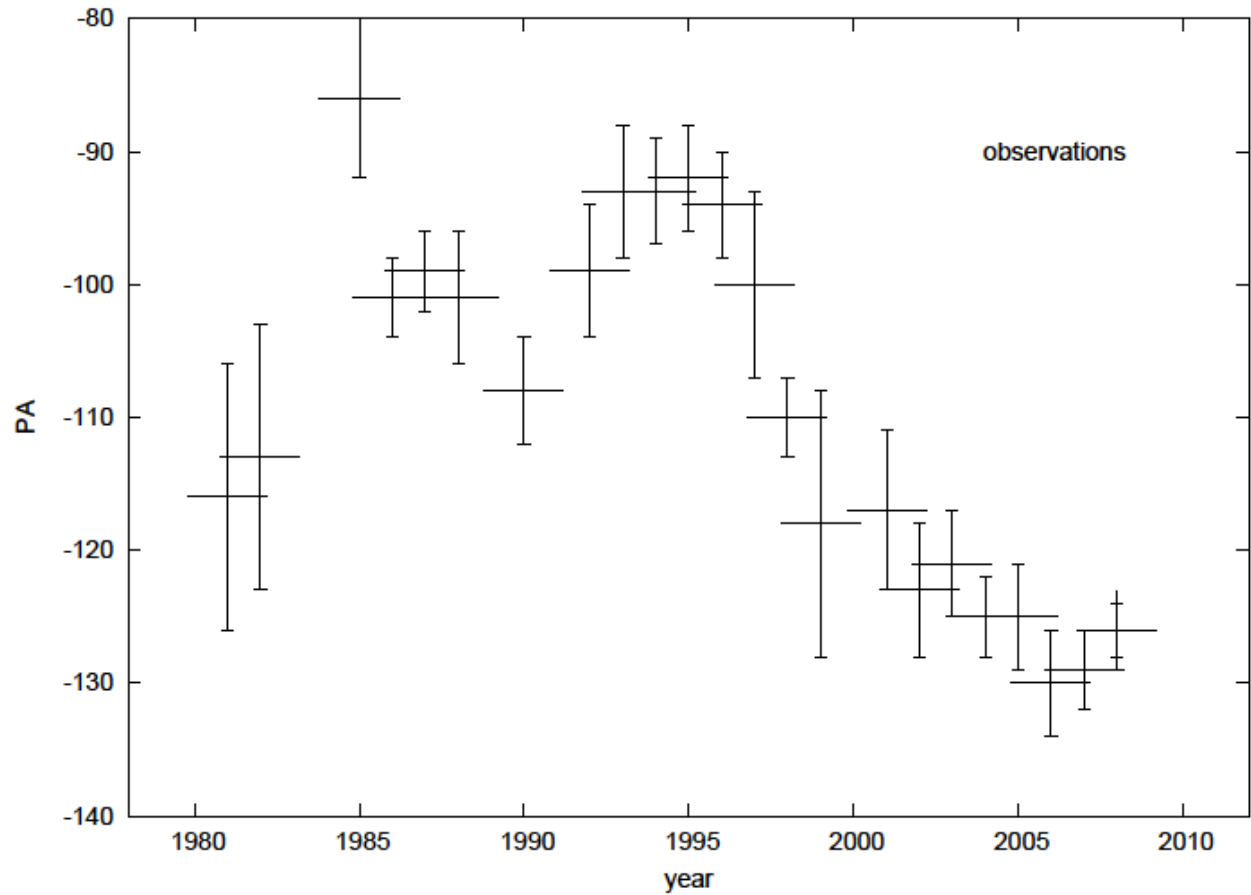
10 Department of Physics, Shoumen University, 9700 Shoumen, Bulgaria

11 Warkauden Kassiopeia ry, Härkämäentie 88, 79480 Kangaslampi, Finland

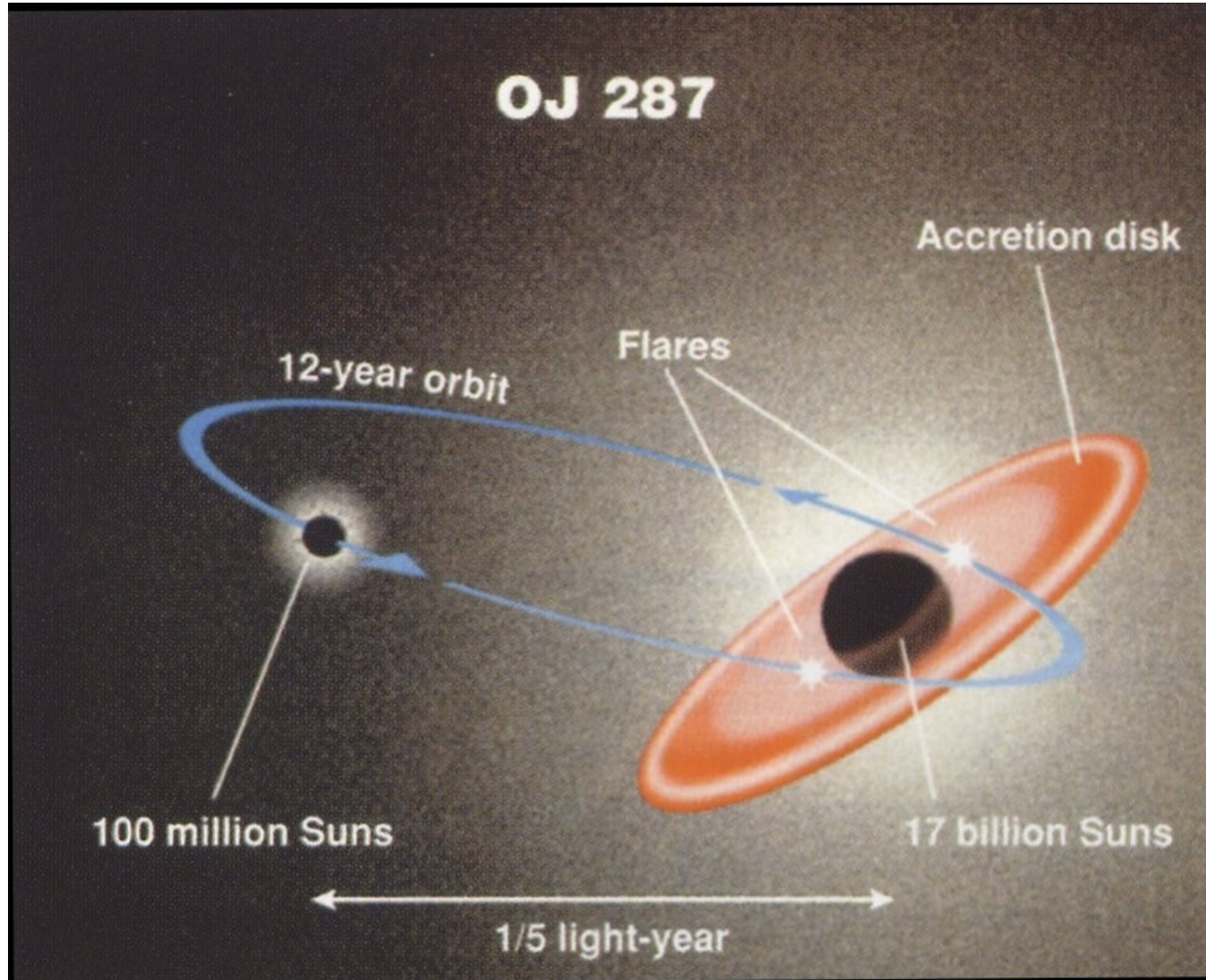
12 Department of Astrophysics, Astronomy and Mechanics, Faculty of Physics, University of Athens, Panepistimiopolis, GR-15784 Zografos, Athens, Greece

13 Canakkale Onsekiz Mart University, Faculty of Physics, TR-17020 Canakkale, Turkey

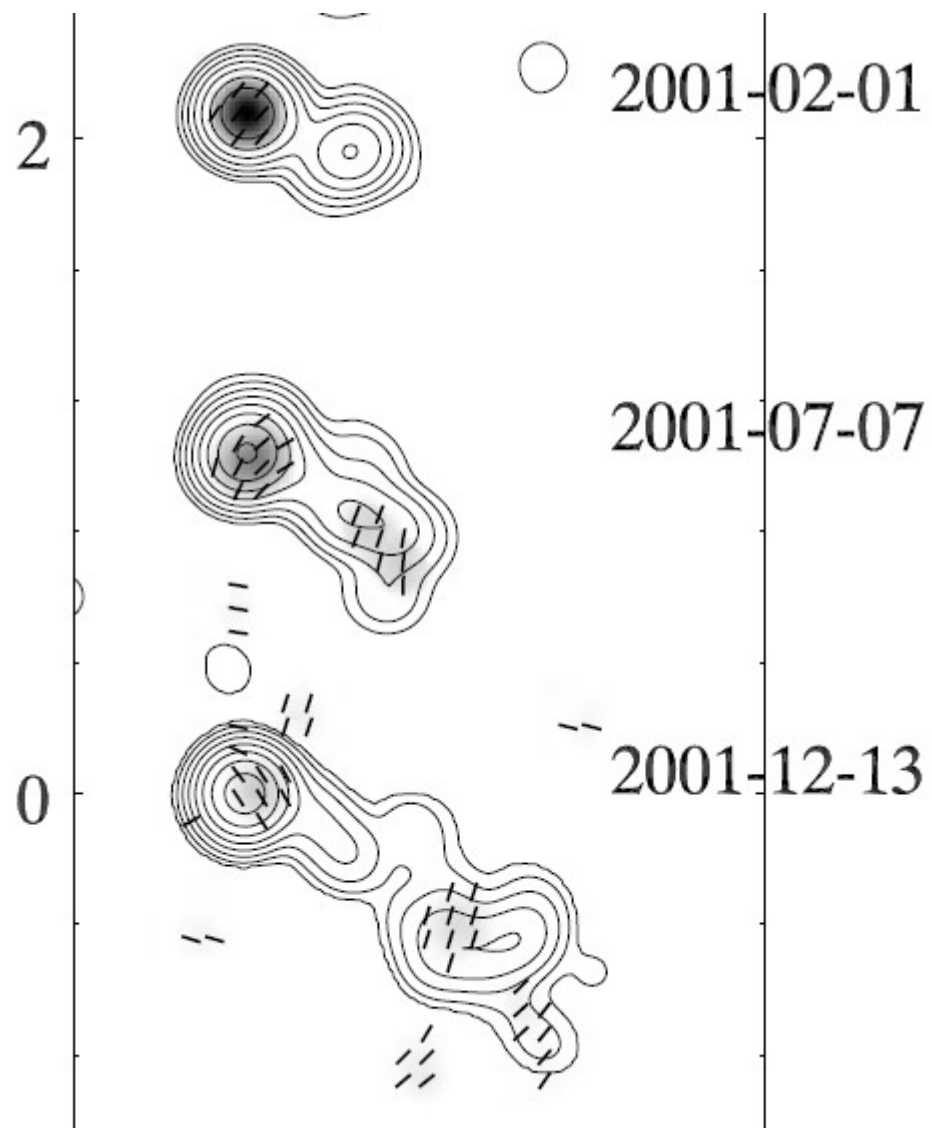
4. Rotation of radio jet



Binary model



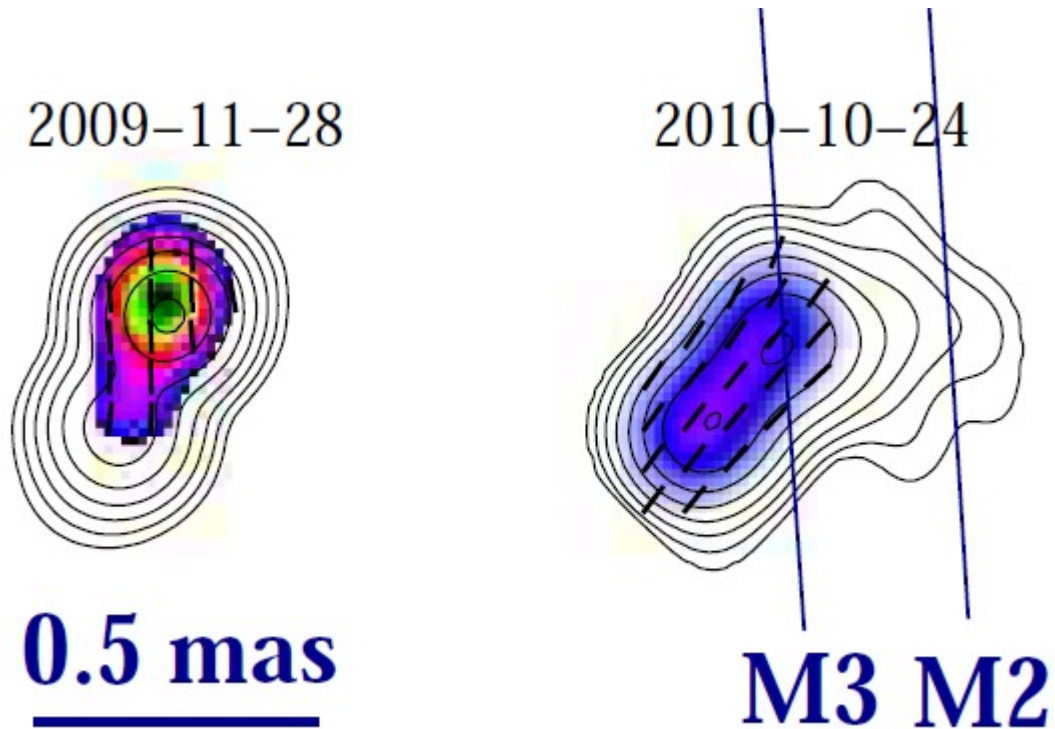




LOCATION OF γ -RAY FLARE EMISSION IN THE JET OF THE BL LACERTAE OBJECT OJ287 MORE THAN 14 PC FROM THE CENTRAL ENGINE

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Accepted for publication in The Astrophysical Journal Letters



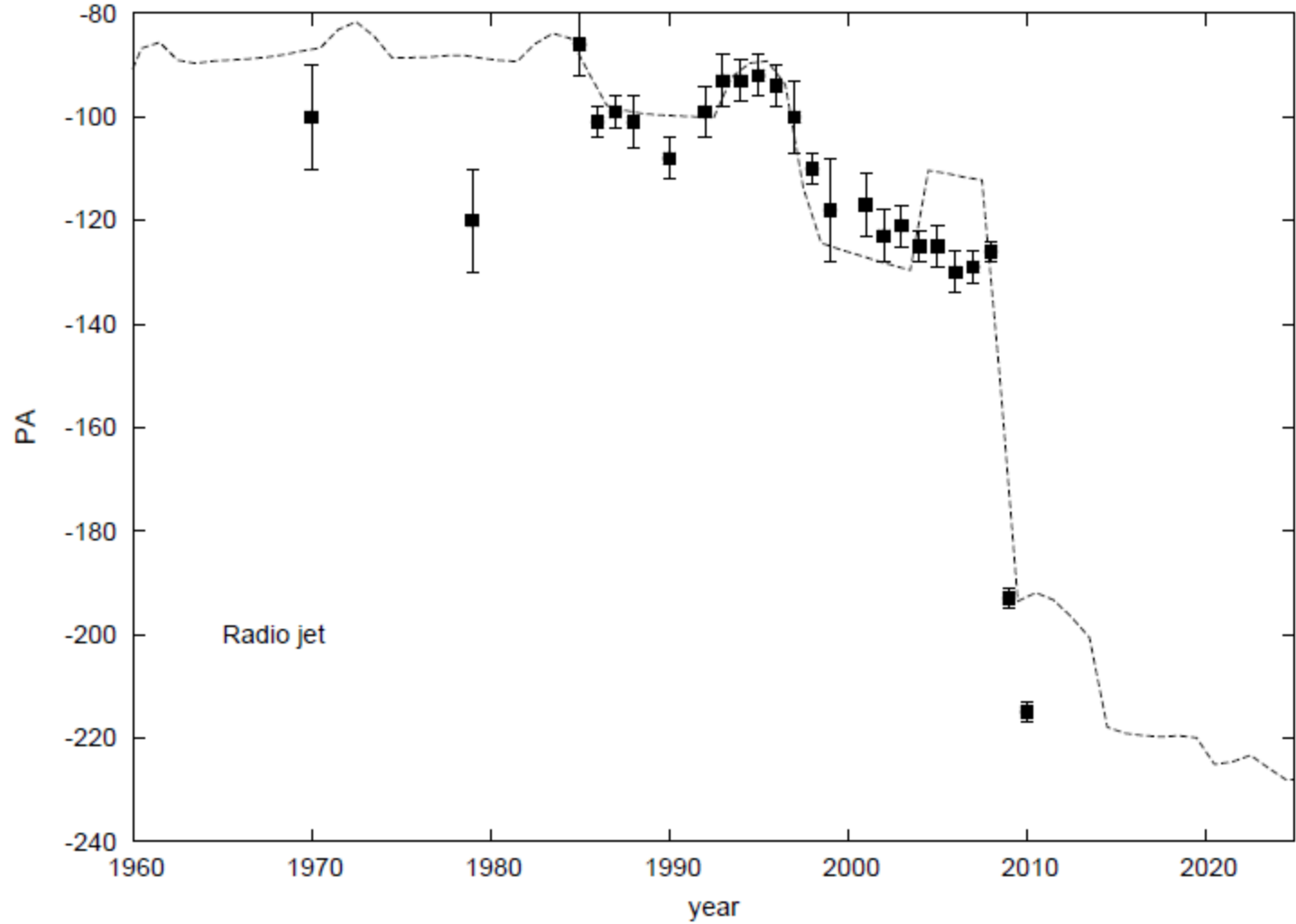


Figure 4. The variation of the position angle of the radio jet in

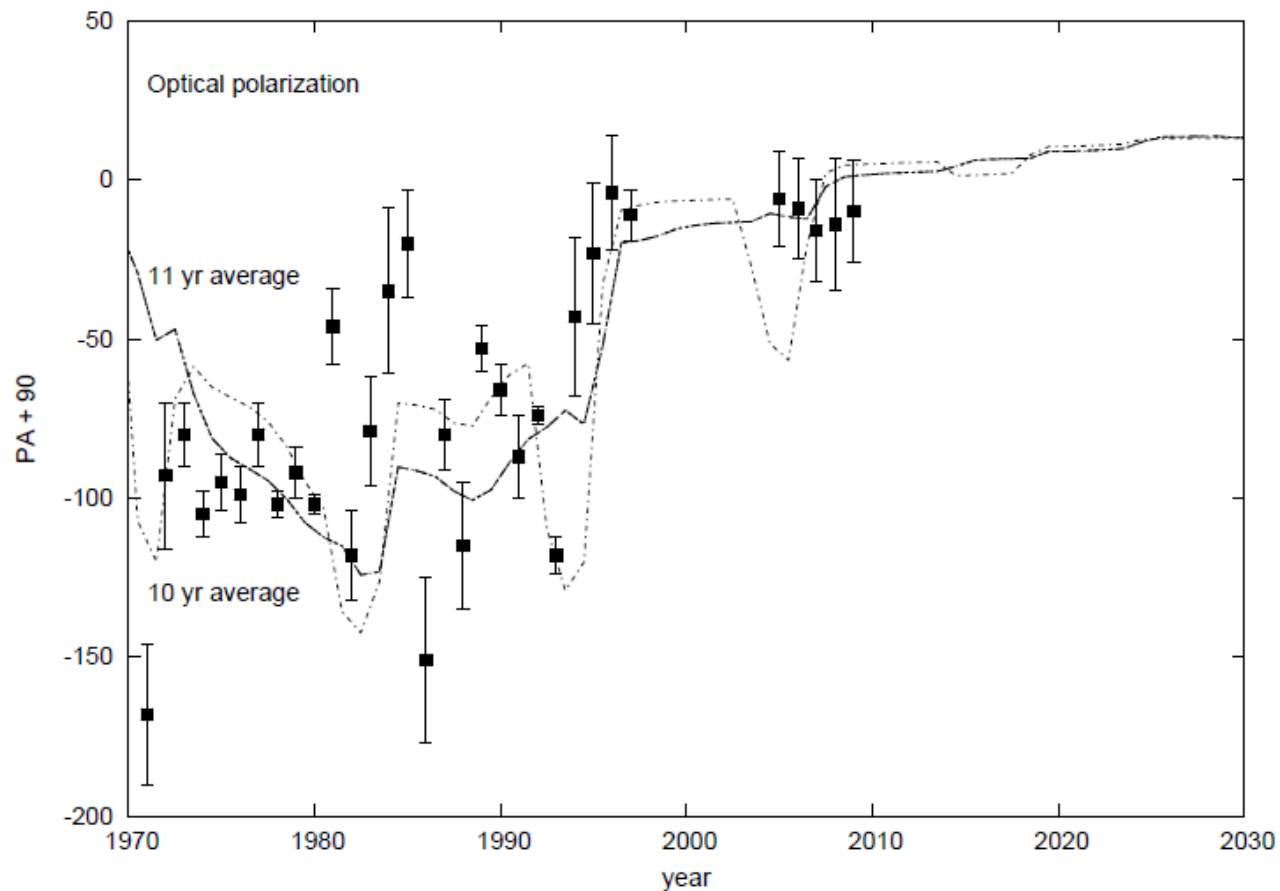
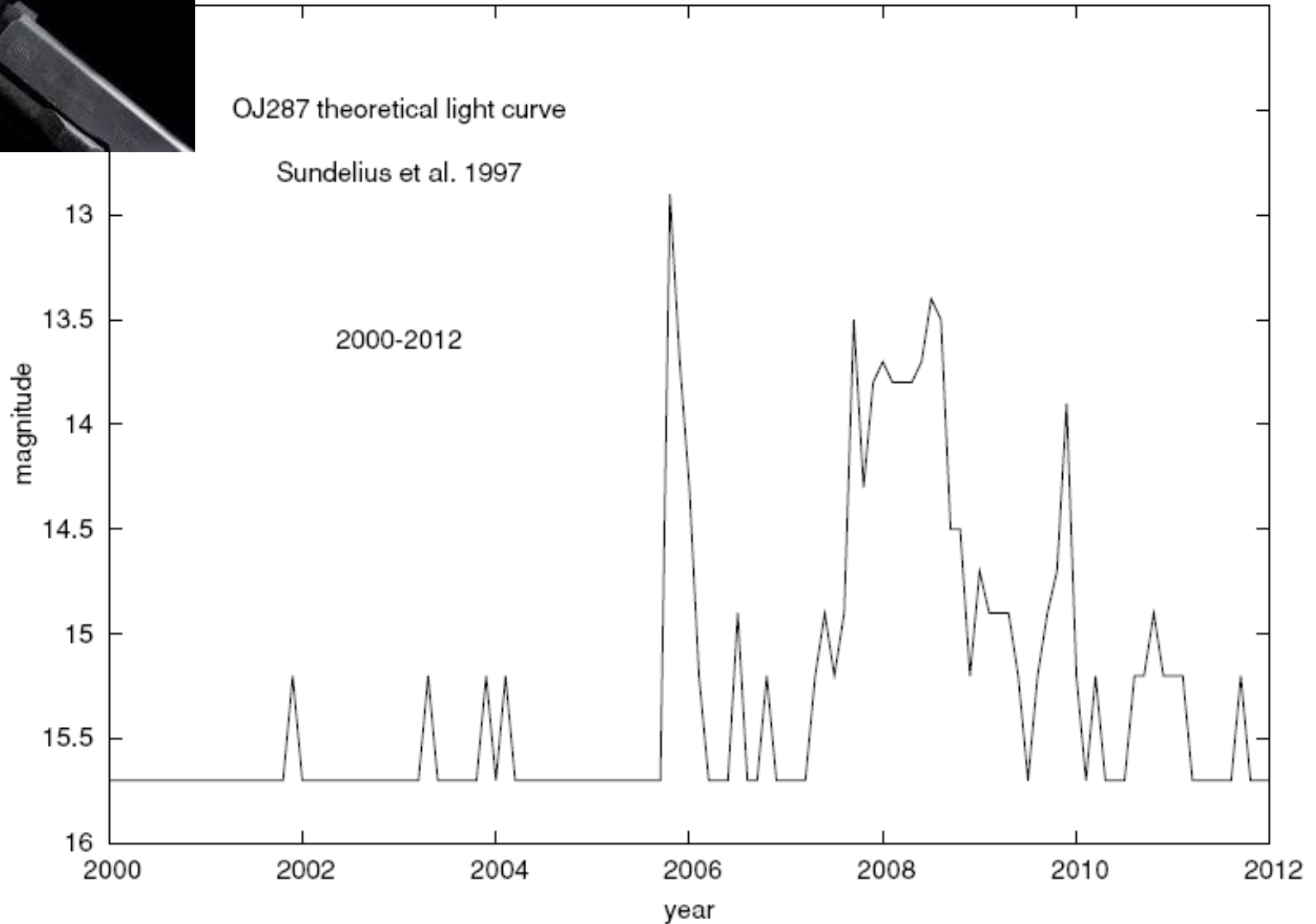
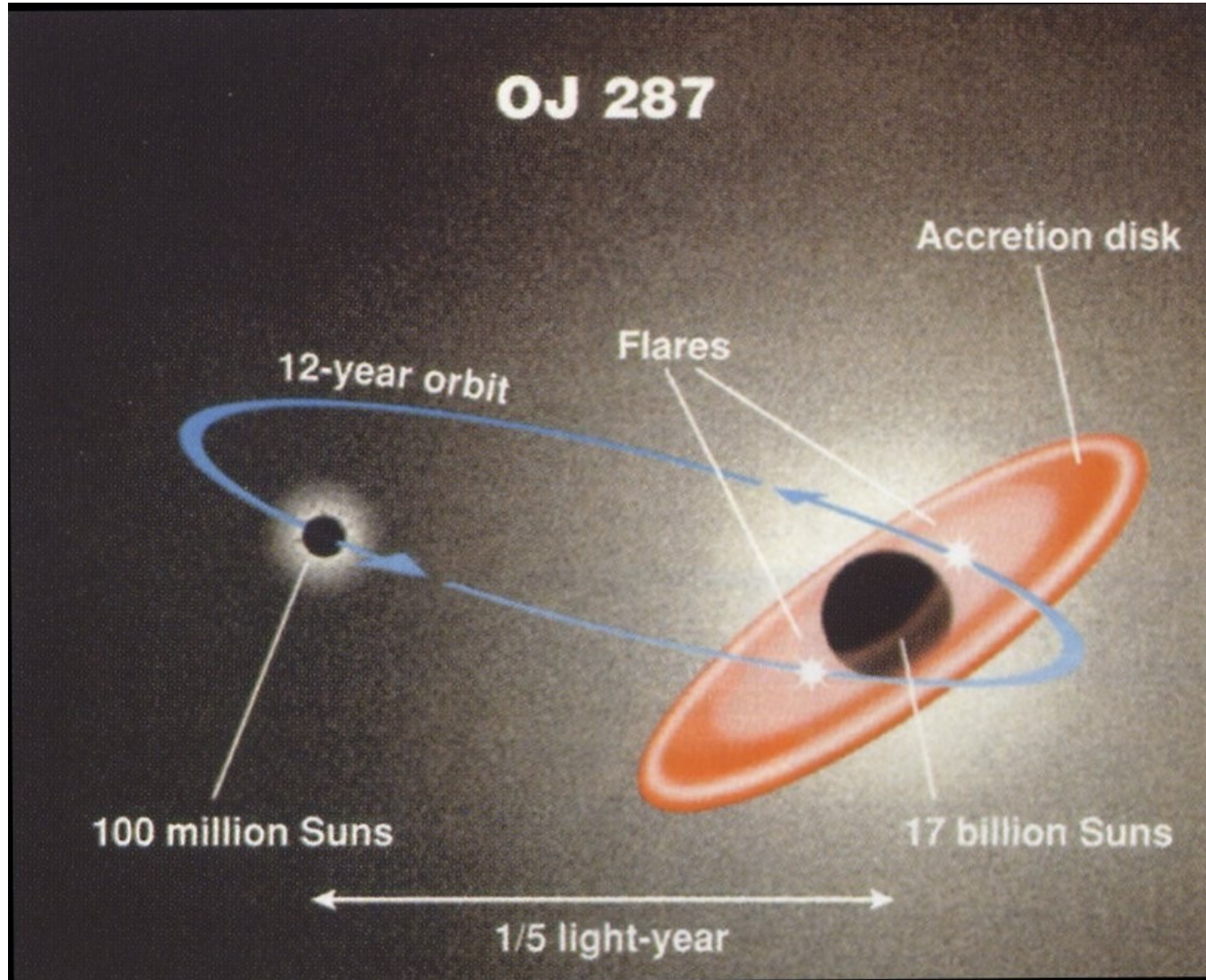


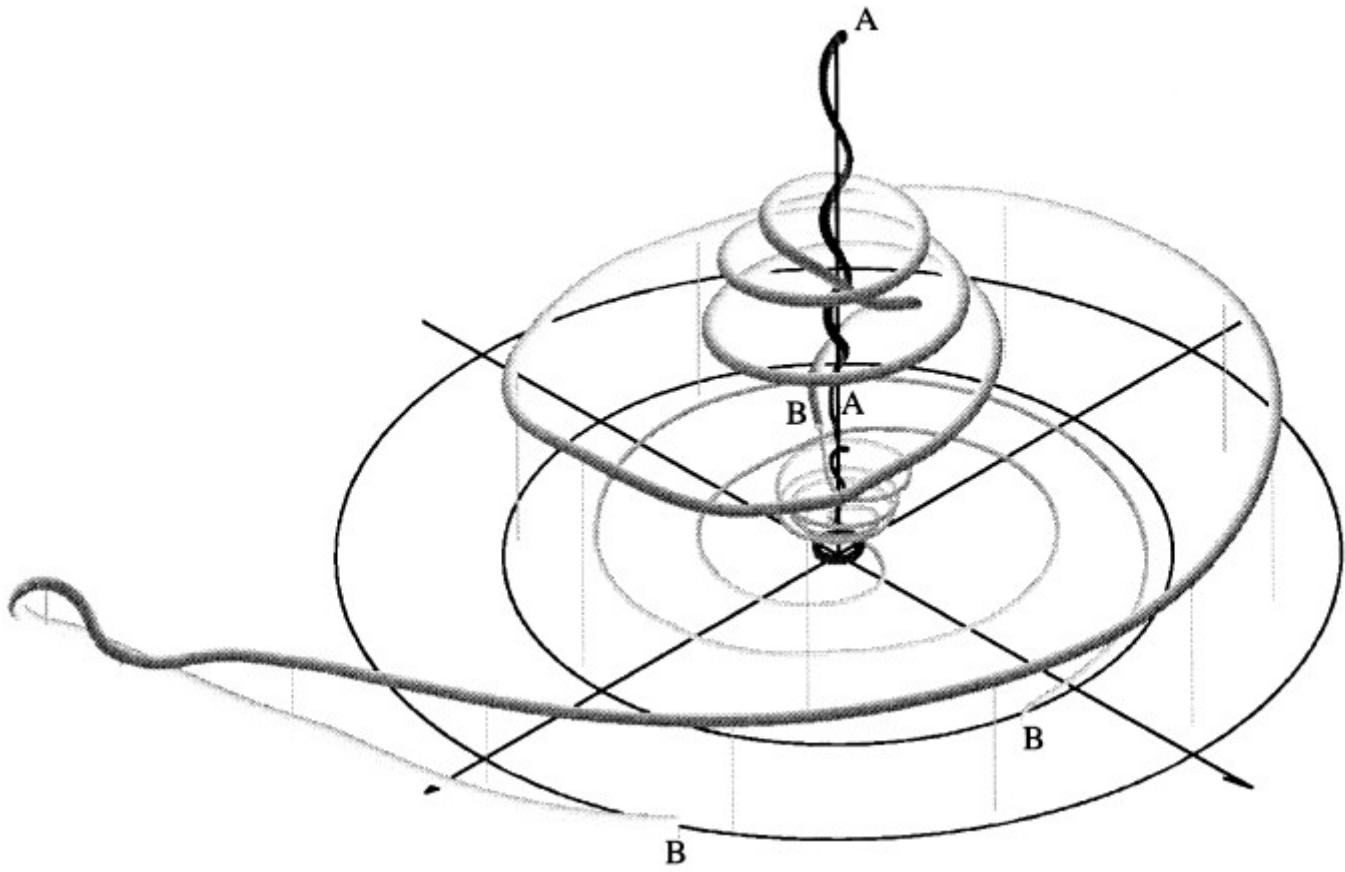
Figure 6. The evolution of the optical polarization angle compared with the model. The curves are for a 10 yr (dash-dot line) and for an 11 yr (dashed line) average. Observations refer to the values in Table 1 minus 180° . The theoretical lines are shifted by adding 90° since the electric vector does not project parallel to the jet axis.

5. Tidal outbursts



Binary model





Testing the 1995 binary black hole model of OJ287

M. J. Valtonen^{1,2}, H. J. Lehto¹, L. O. Takalo¹, A. Sillanpää¹

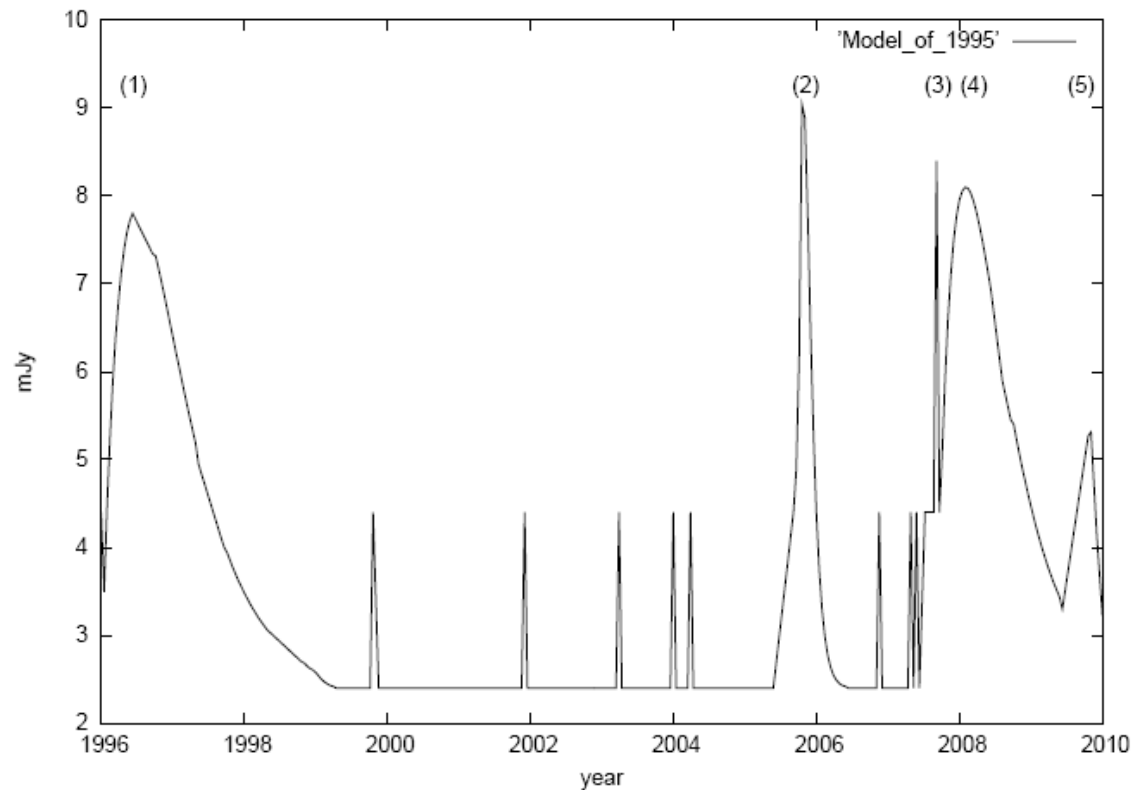


Fig. 1.— The 1995 light curve of OJ287. Five ma-

Observed optical brightness 1996-2010

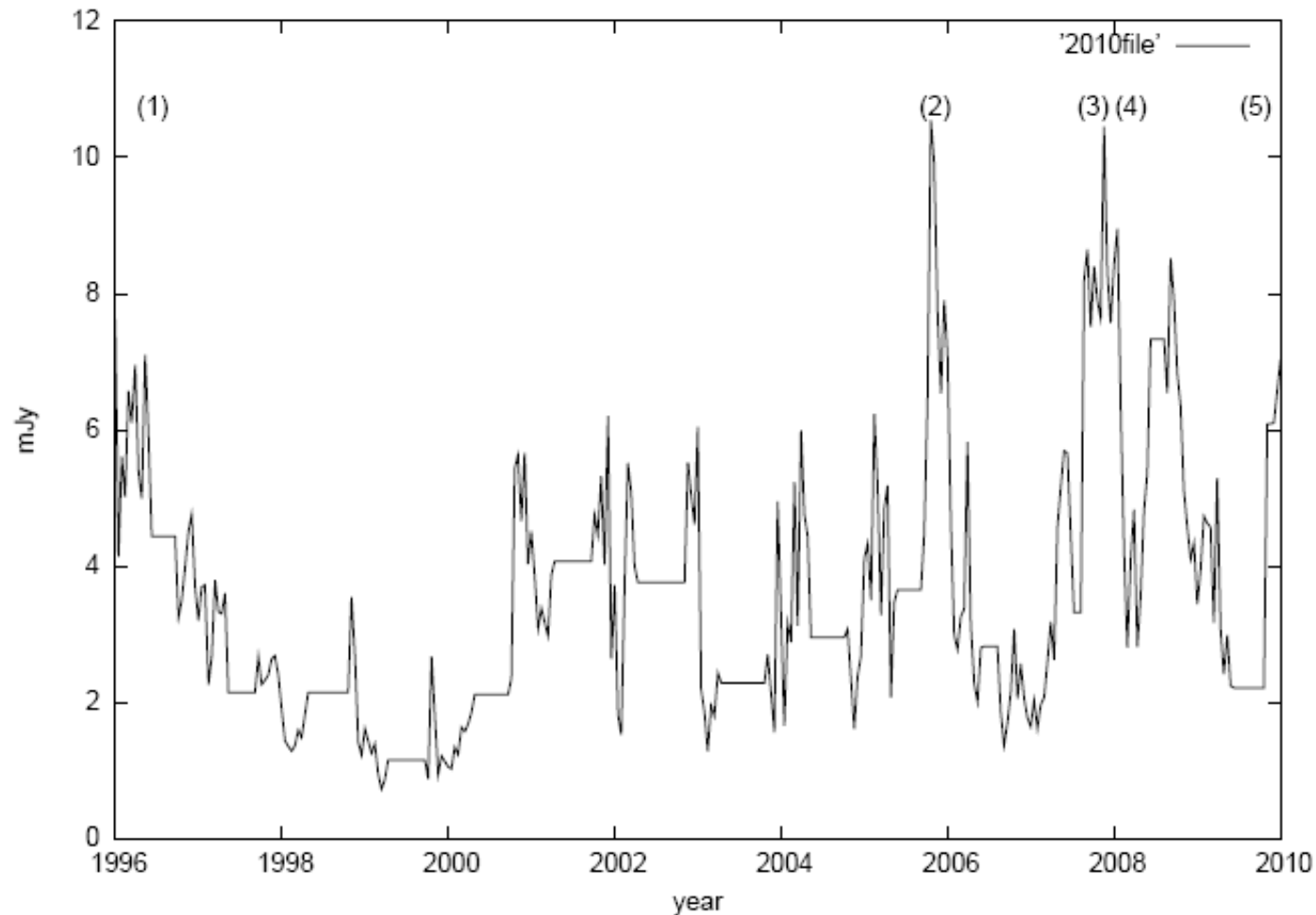


Fig. 2.— The observations of OJ287 during 1996

Residuals: quasi-periodic oscillations: probability $< 10^{-8}$

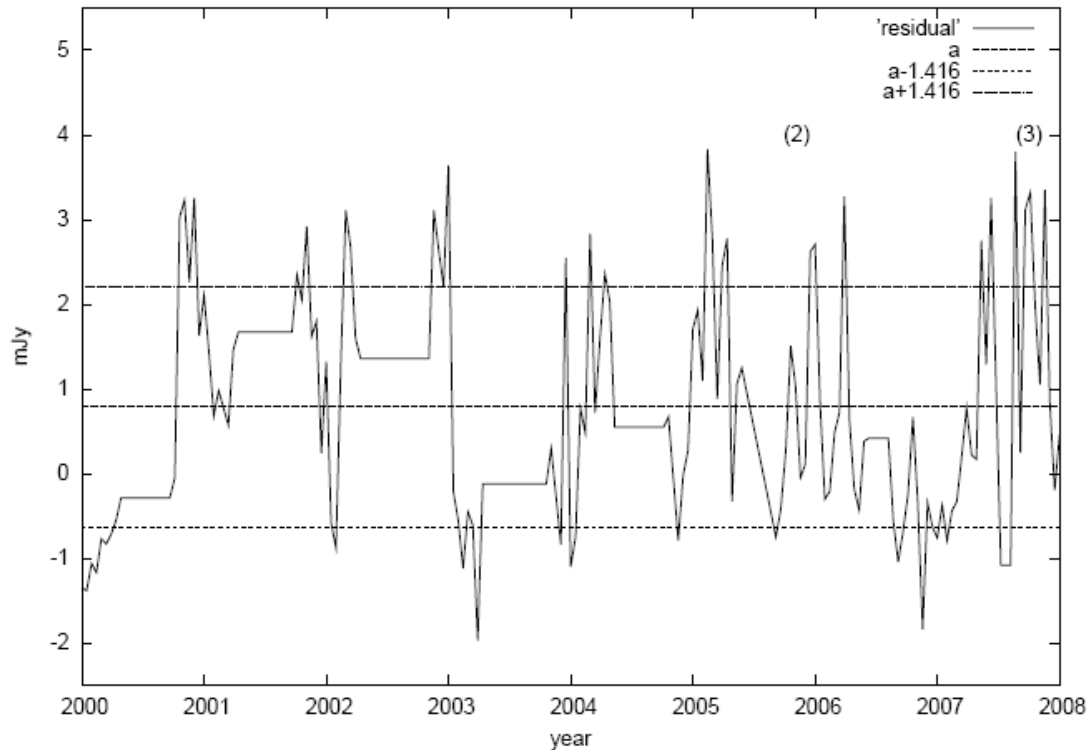


Fig. 3.— The residuals observations minus the model for OJ287 during the 2000 - 2008.



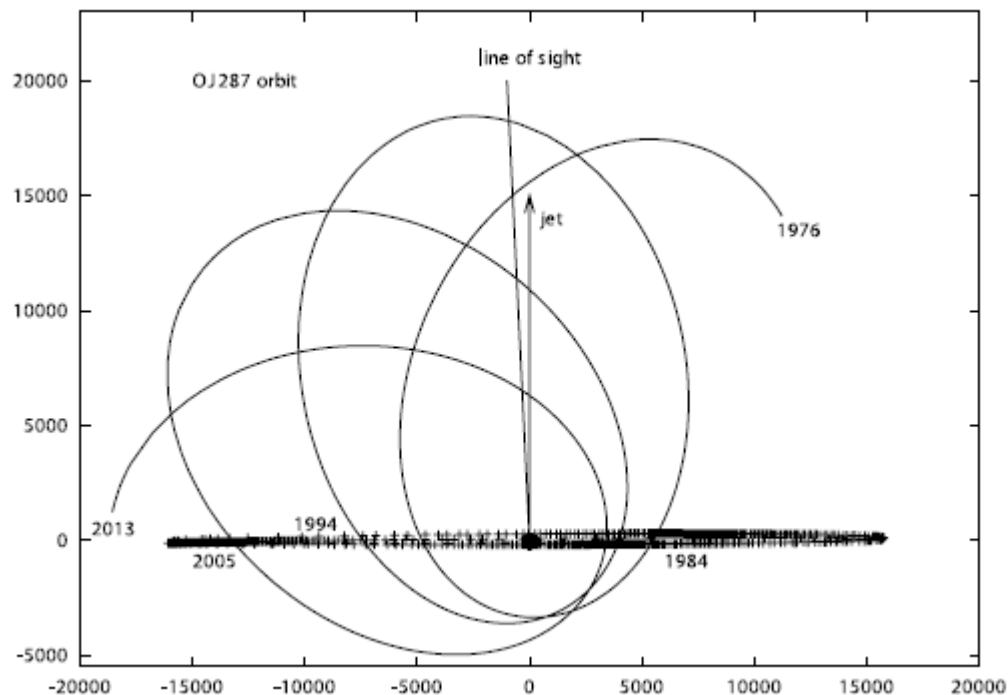
Next: 2015 - 2022

campaign

2015: accurate spin value

2019: test of no-hair -theorem

2022: confirmation of precession



PN acceleration

$$\ddot{\mathbf{x}} \equiv \frac{d^2 \mathbf{x}}{dt^2} = \ddot{\mathbf{x}}_0 + \ddot{\mathbf{x}}_{1\text{PN}} + \ddot{\mathbf{x}}_{\text{SO}} + \ddot{\mathbf{x}}_Q$$
$$+ \ddot{\mathbf{x}}_{2\text{PN}} + \ddot{\mathbf{x}}_{2.5\text{PN}},$$
$$\ddot{\mathbf{x}}_0 = -\frac{Gm}{r^3} \mathbf{x}$$

$$\ddot{\mathbf{x}}_{1\text{PN}} = -\frac{Gm}{c^2 r^2} \left\{ \mathbf{n} \left[-2(2 + \eta) \frac{Gm}{r} + (1 + 3\eta)v^2 - \frac{3}{2}\eta\dot{r}^2 \right] - 2(2 - \eta)\dot{r}\mathbf{v} \right\},$$

$$\begin{aligned} \ddot{\mathbf{x}}_{2\text{PN}} = & -\frac{Gm}{c^4 r^2} \left\{ \mathbf{n} \left[\frac{3}{4}(12 + 29\eta) \left(\frac{Gm}{r} \right)^2 \right. \right. \\ & + \eta(3 - 4\eta)v^4 + \frac{15}{8}\eta(1 - 3\eta)\dot{r}^4 \\ & - \frac{3}{2}\eta(3 - 4\eta)v^2\dot{r}^2 - \frac{1}{2}\eta(13 - 4\eta) \left(\frac{Gm}{r} \right) v^2 \\ & \left. \left. - (2 + 25\eta + 2\eta^2) \left(\frac{Gm}{r} \right) \dot{r}^2 \right] \right. \\ & - \frac{1}{2}\dot{r}\mathbf{v} \left[\eta(15 + 4\eta)v^2 - (4 + 41\eta + 8\eta^2) \left(\frac{Gm}{r} \right) \right. \\ & \left. \left. - 3\eta(3 + 2\eta)\dot{r}^2 \right] \right\}, \end{aligned}$$

Spin-orbit term

$$\begin{aligned} \ddot{\mathbf{x}}_{\text{SO}} = & \frac{G m}{r^2} \left(\frac{G m}{c^3 r} \right) \left(\frac{1 + \sqrt{1 - 4\eta}}{4} \right) \\ & \times \chi \left\{ \left[12 [\mathbf{s}_1 \cdot (\mathbf{n} \times \mathbf{v})] \right] \mathbf{n} \right. \\ & + \left[\left(9 + 3\sqrt{1 - 4\eta} \right) \dot{r} \right] (\mathbf{n} \times \mathbf{s}_1) \\ & \left. - \left[7 + \sqrt{1 - 4\eta} \right] (\mathbf{v} \times \mathbf{s}_1) \right\}, \end{aligned}$$

$$\ddot{\mathbf{x}}_{2.5\text{PN}} = \frac{8}{15} \frac{G^2 m^2 \eta}{c^5 r^3} \left\{ \left[9v^2 + 17 \frac{Gm}{r} \right] \dot{r} \mathbf{n} - \left[3v^2 + 9 \frac{Gm}{r} \right] \mathbf{v} \right\},$$

$$\ddot{\mathbf{x}}_Q = q \chi^2 \frac{3G^3 m_1^2 m}{2c^4 r^4} \left\{ \left[5(\mathbf{n} \cdot \mathbf{s}_1)^2 - 1 \right] \mathbf{n} - 2(\mathbf{n} \cdot \mathbf{s}_1) \mathbf{s}_1 \right\}$$

no-hair theorem

quadrupole moment Q

$$Q = -q \frac{S^2}{Mc^2}$$

$$\mathbf{S}_1 = G m_1^2 \chi \mathbf{s}_1 / c$$

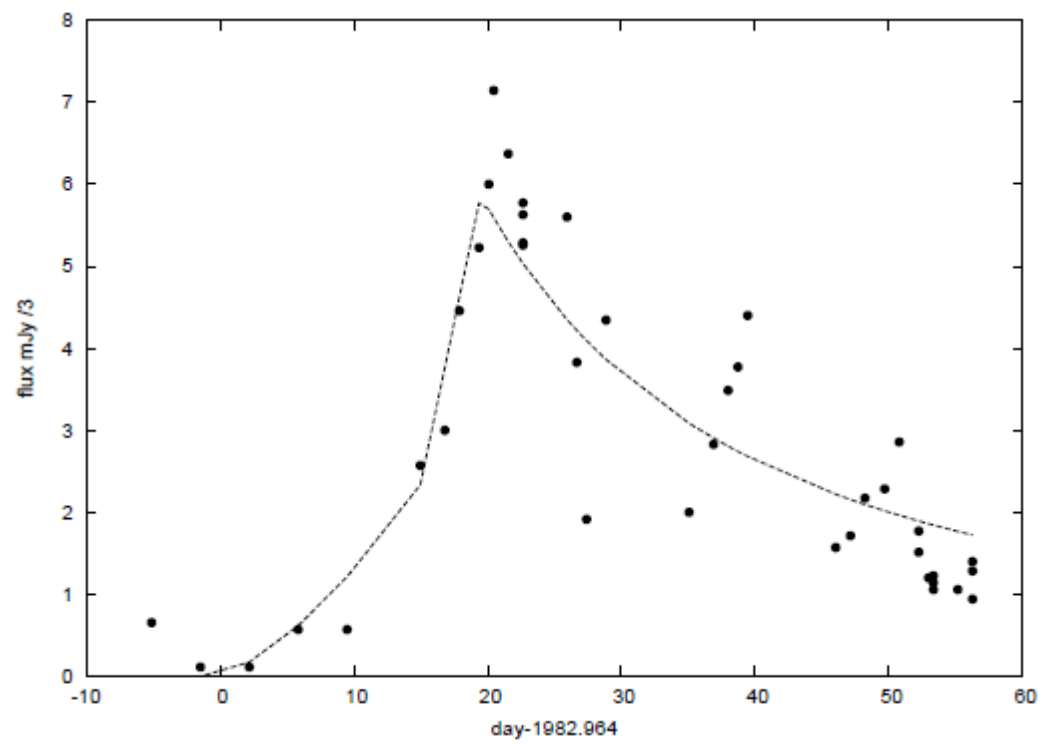
unit vector \mathbf{s}_1

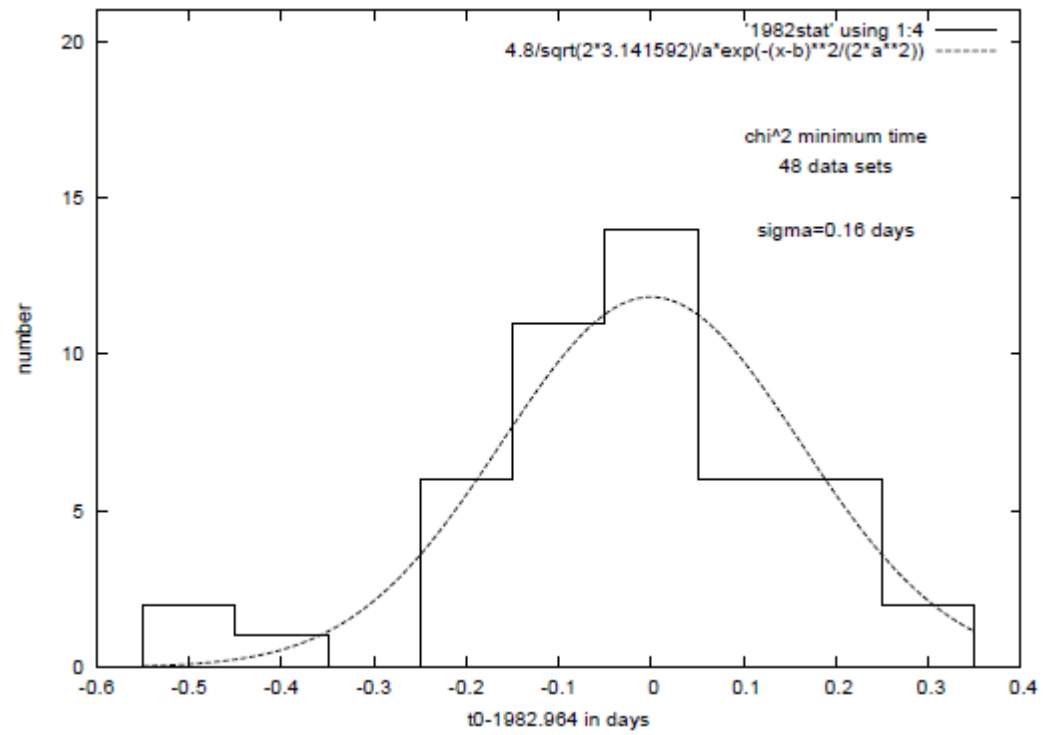
For black holes $q = 1$

for neutron stars $q > 2$

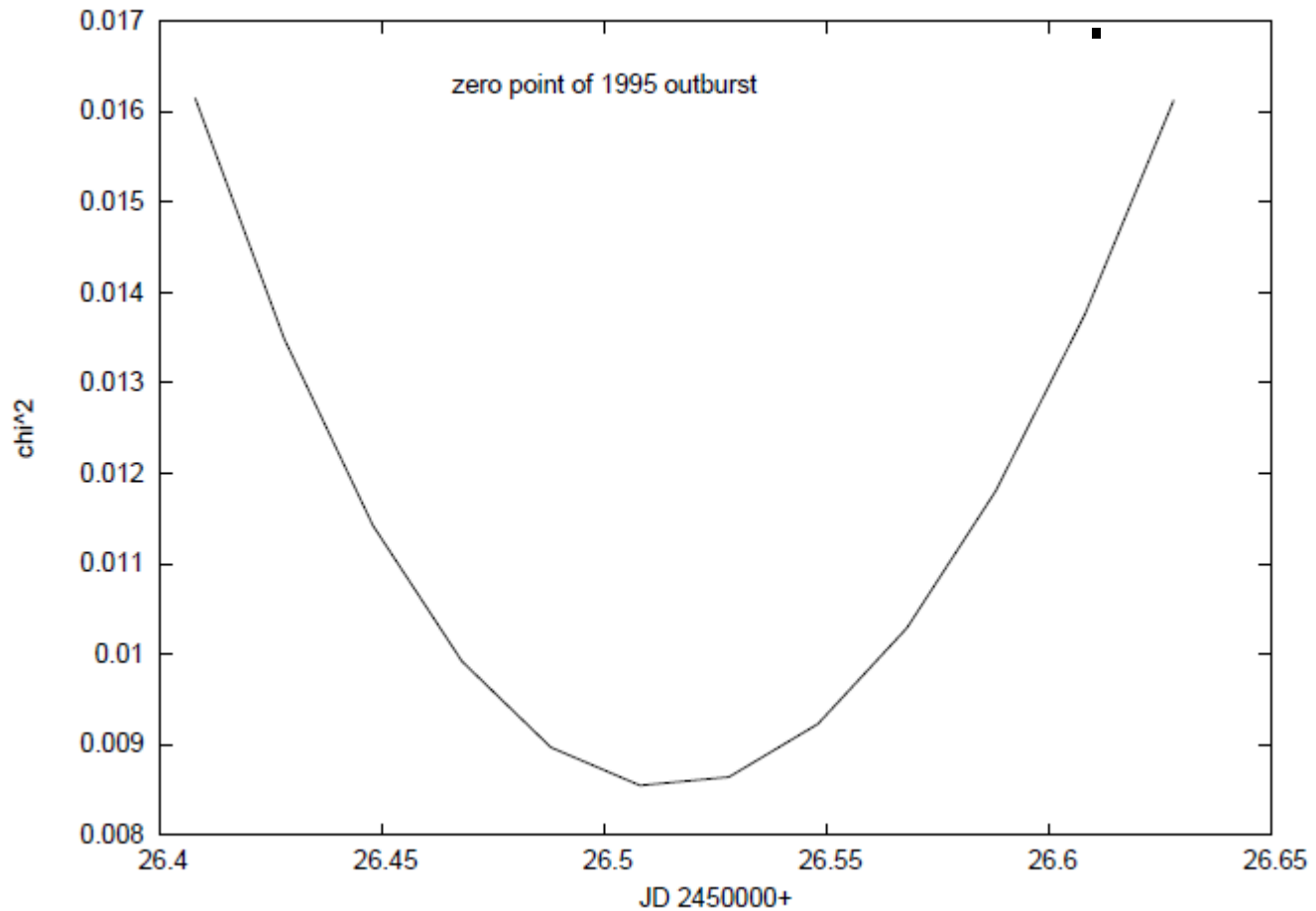
Parameters

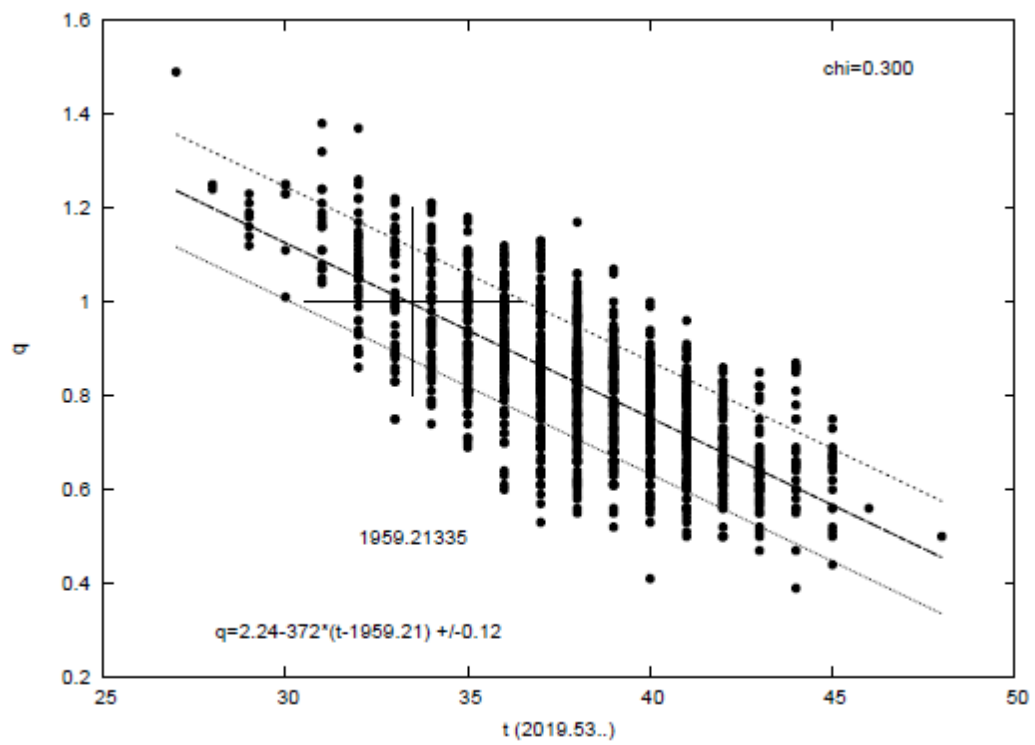
$\Delta\phi^\circ$	39.1 ± 0.1
m_1	$(1.83 \pm 0.01) \cdot 10^{10} M_\odot$
m_2	$(1.4 \pm 0.1) \cdot 10^8 M_\odot$
χ	0.28 ± 0.08
ϕ_0	$56^\circ.5 \pm 1^\circ.2$
e_0	0.6584 ± 0.001
q	1.0 ± 0.3
t_d	0.75 ± 0.04

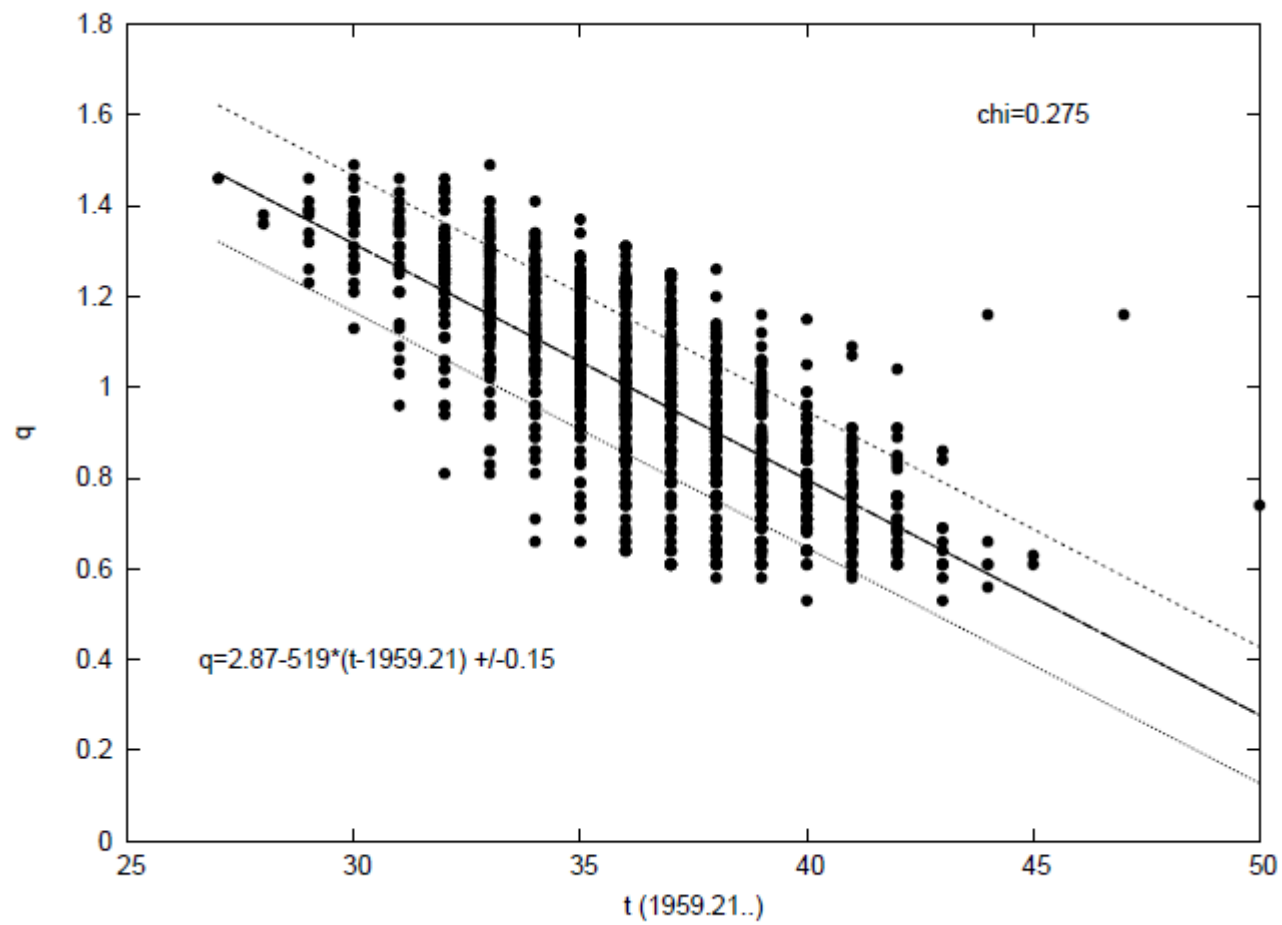




1995.8426







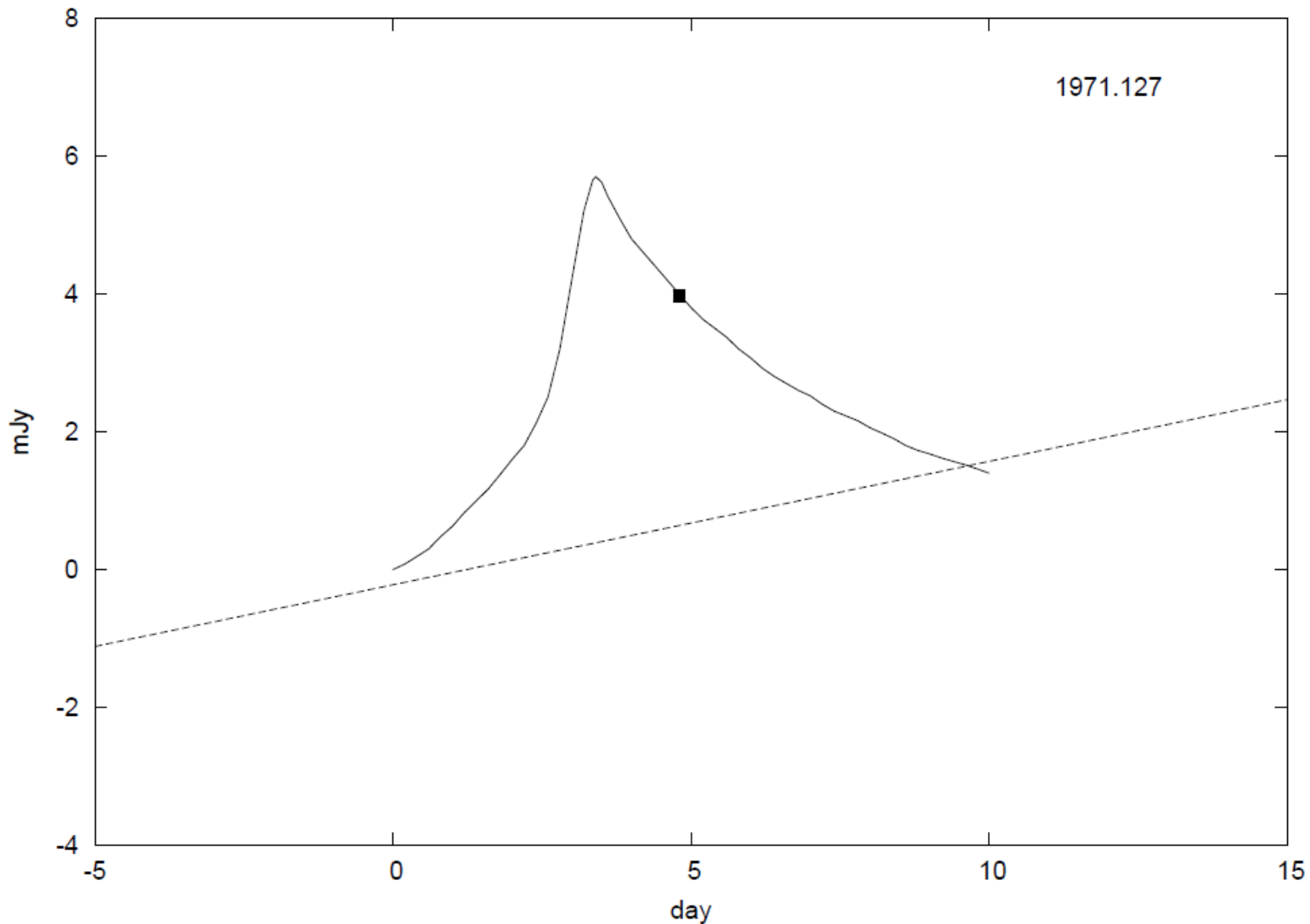
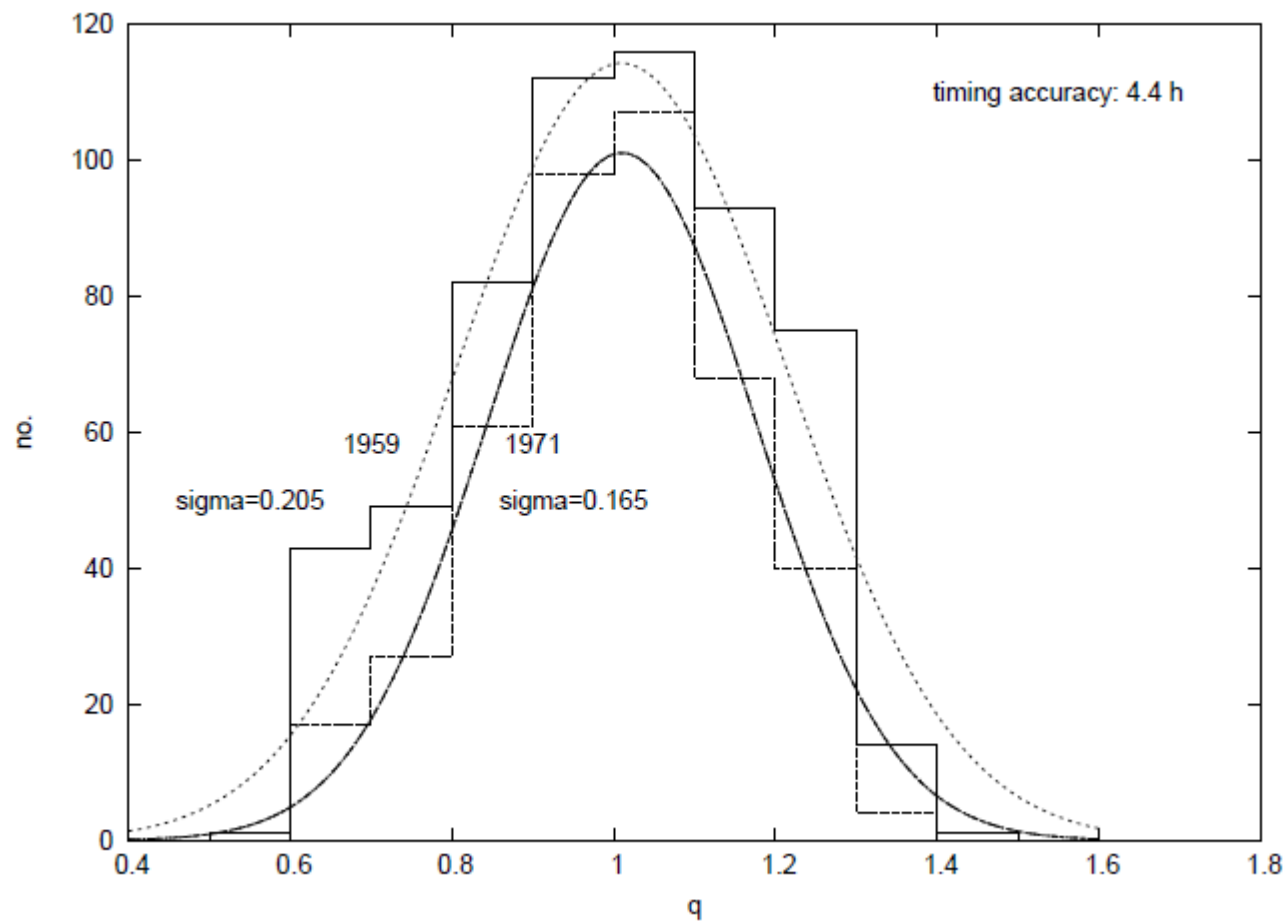
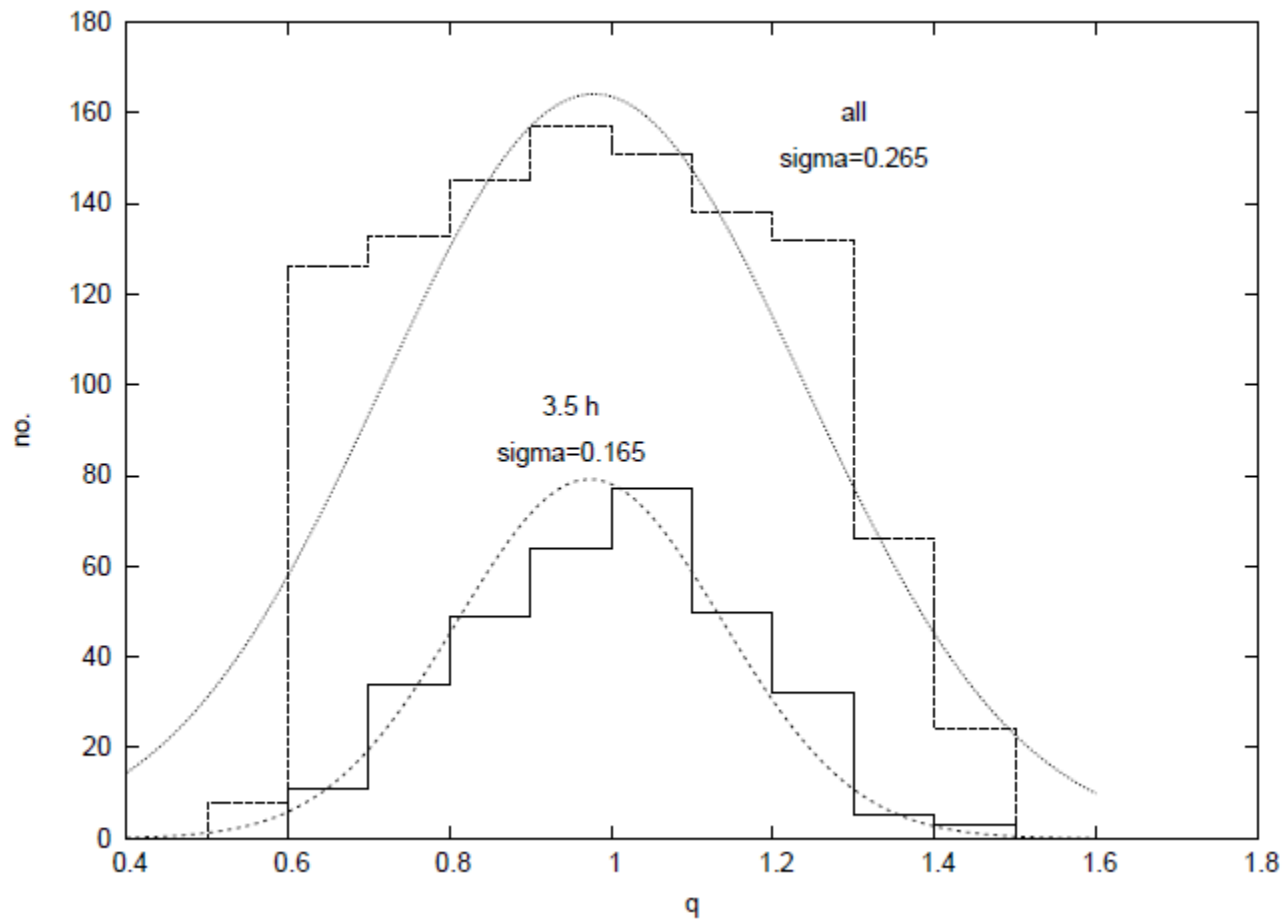


Fig. 2.— Observations of the brightness of OJ287 at the expected 1971 outburst time. The solid

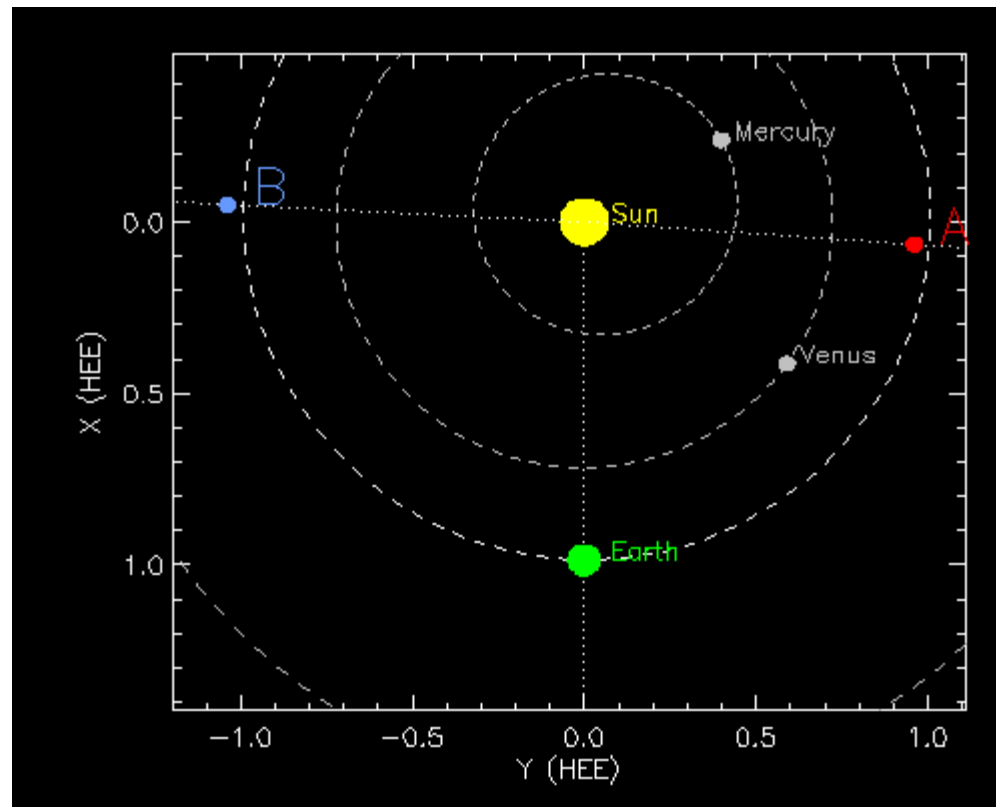


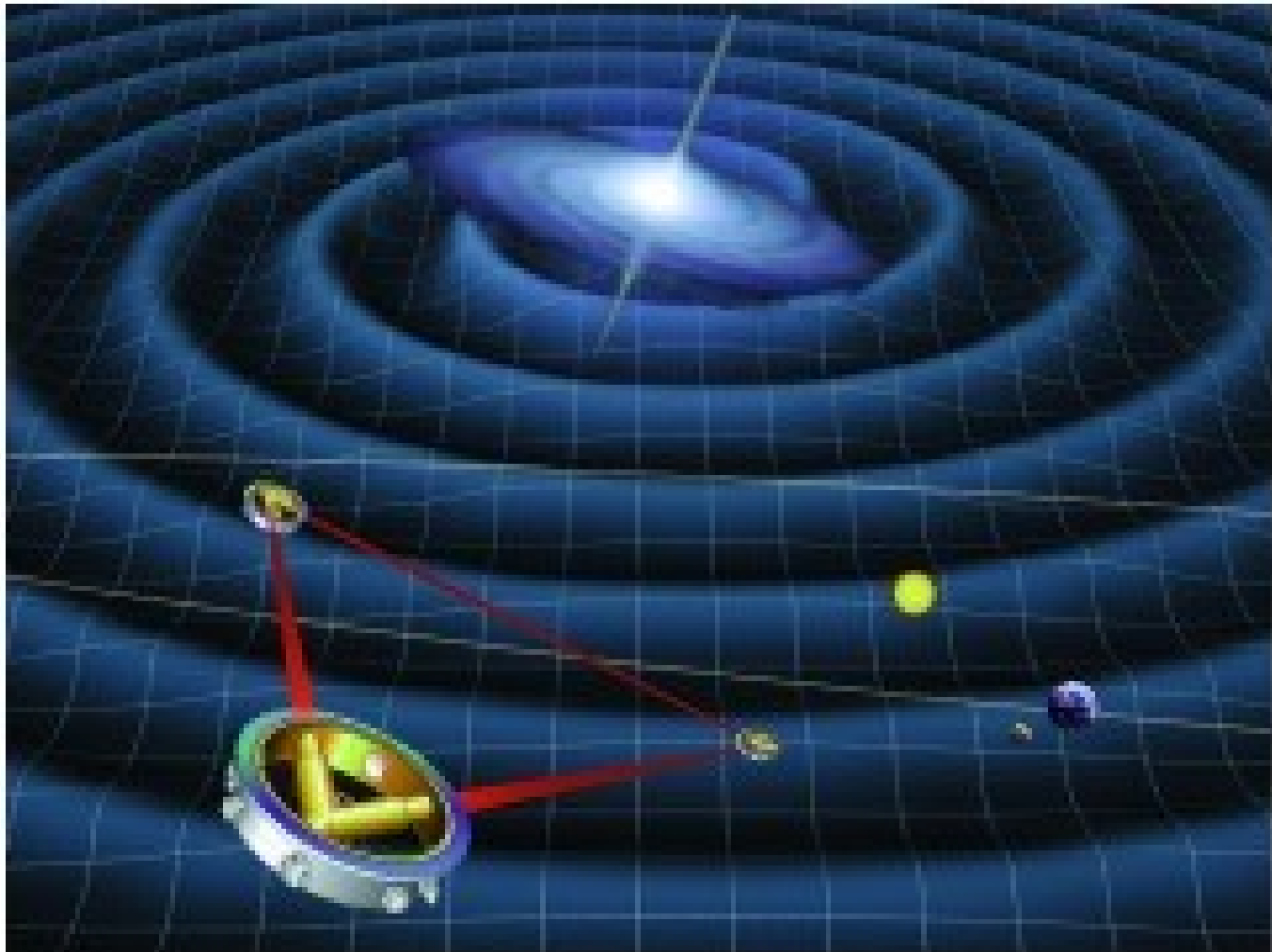


On the possibility of testing black hole no-hair theorem during the present decade with OJ287

M. J. Valtonen^{1,2}, S. Mikkola¹, H. J. Lehto¹, A. Gopakumar³, R. Hudec^{4,5}
and
J. Polednikova^{4,5}

Space-based photometry is required in 2019 July

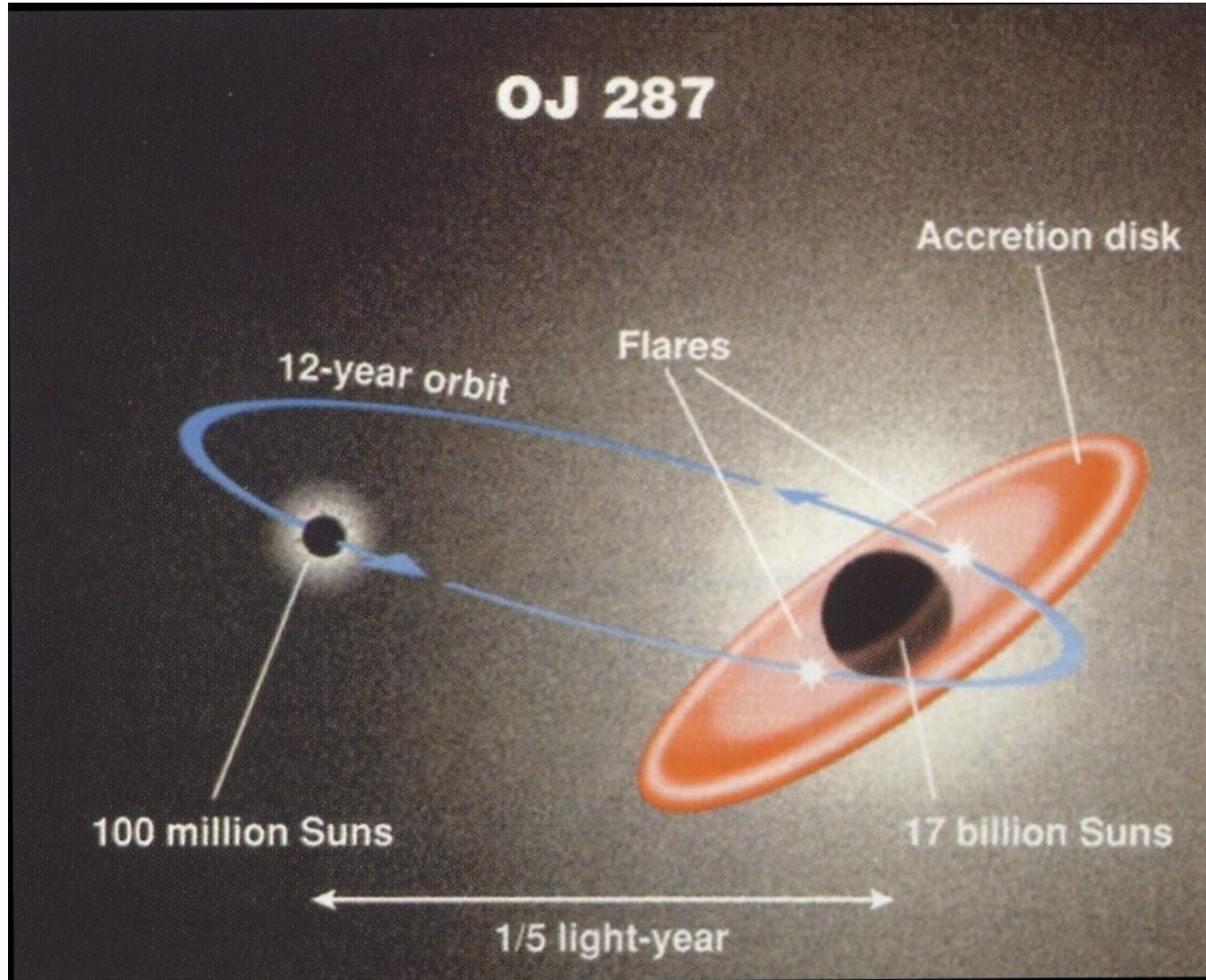




Artist's impression of the LISA spacecraft
© NASA (public domain)

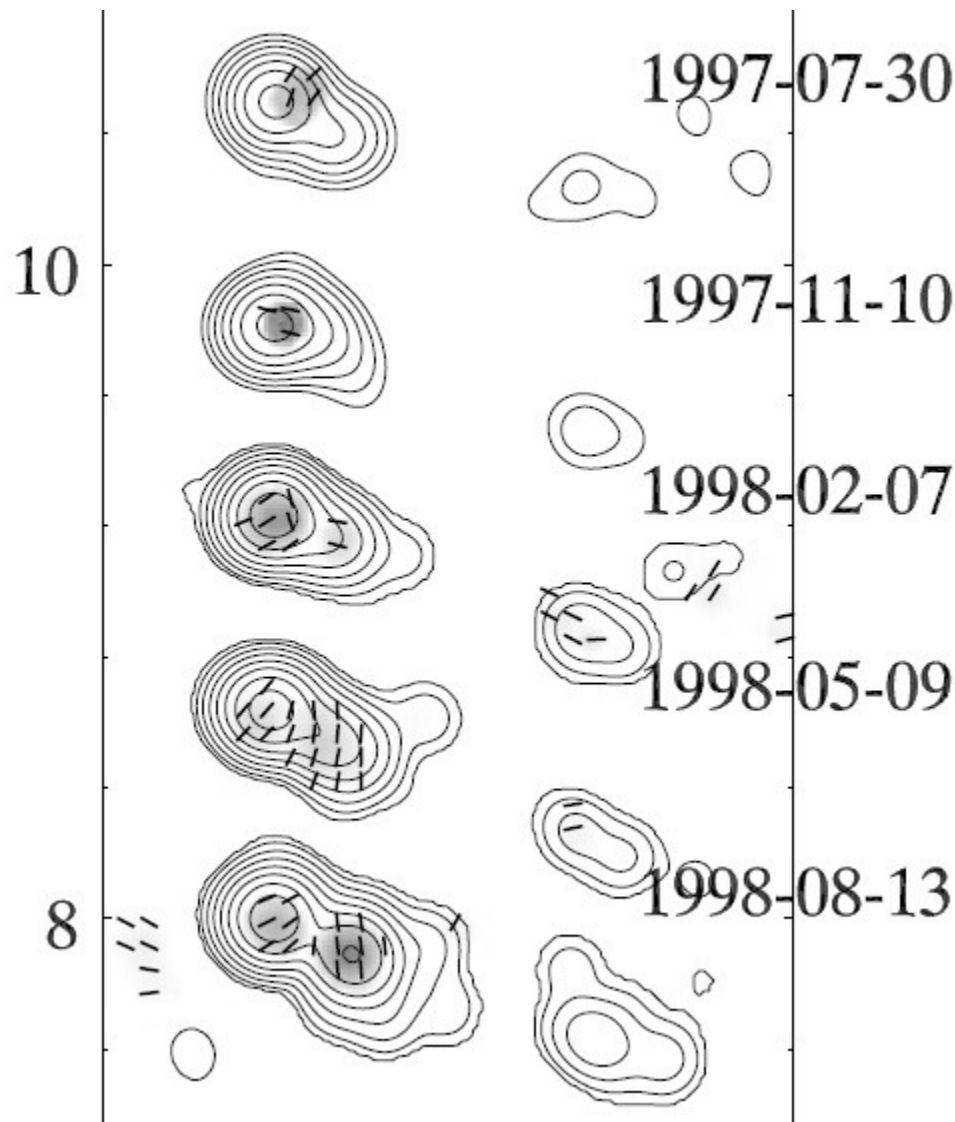


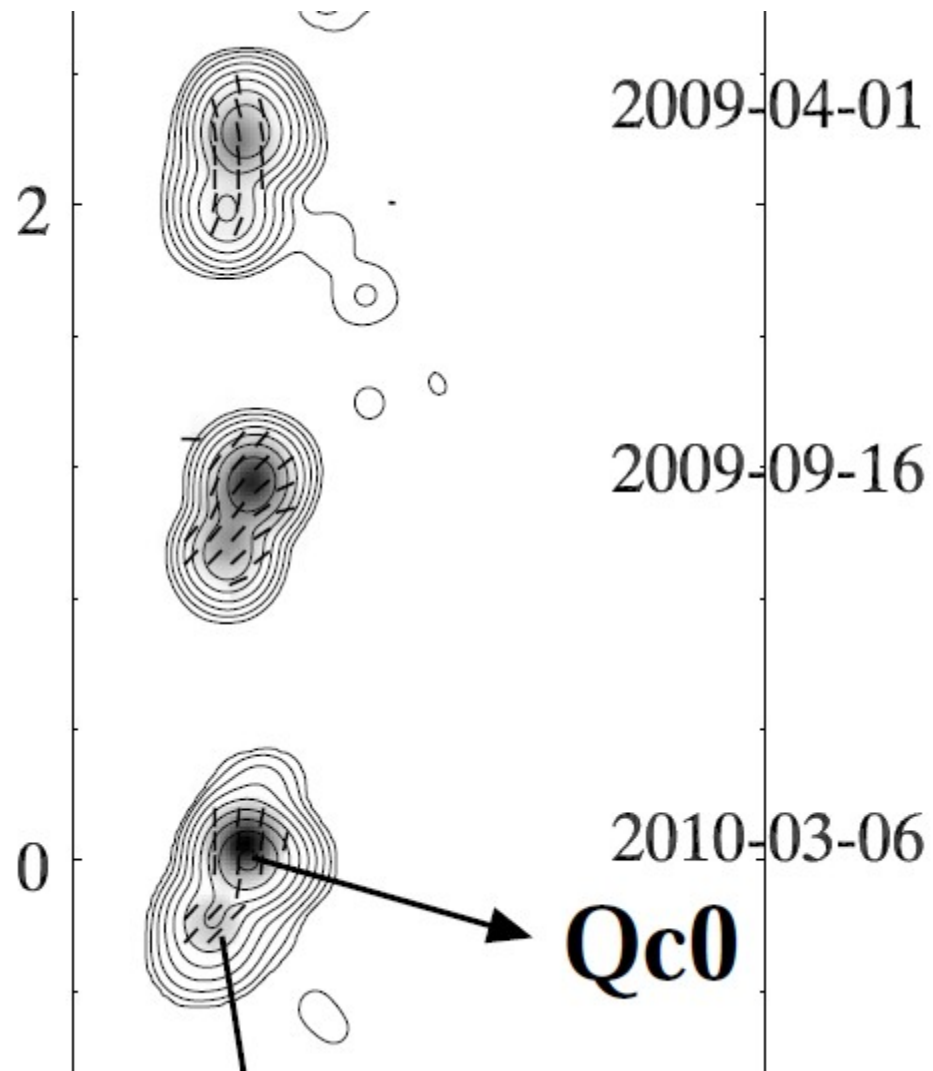
Binary model



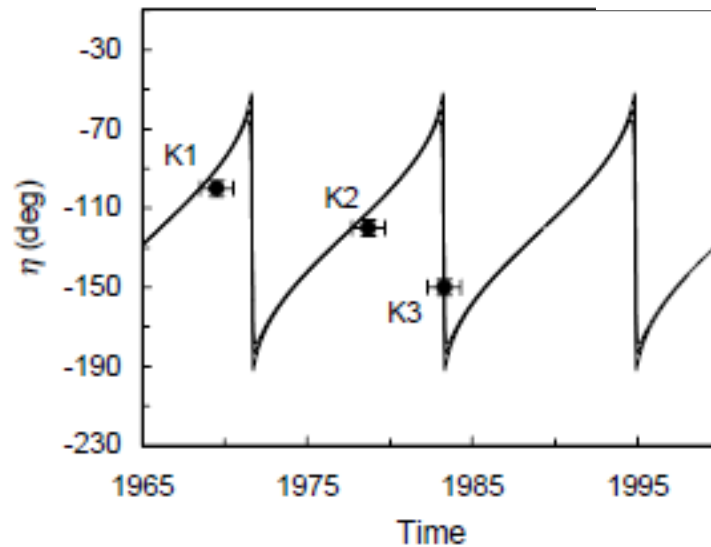
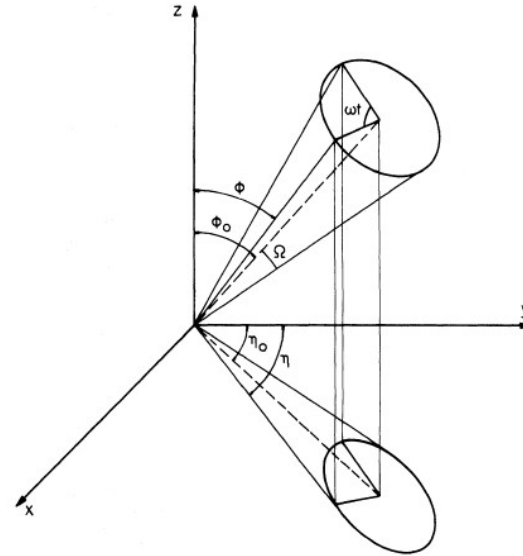
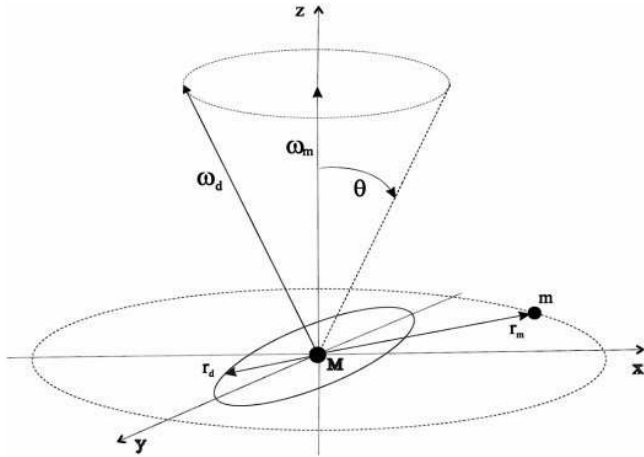
The multi-spectral-range behavior of OJ 287 in 2005–2010

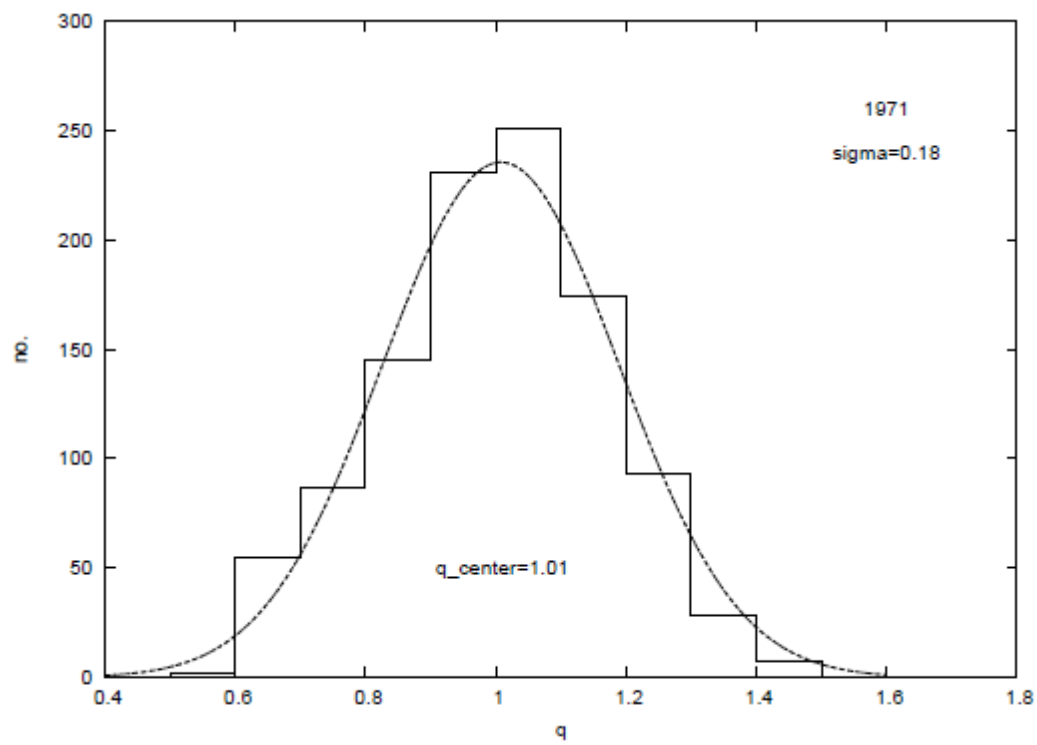
I. Agudo¹, S. G. Jorstad¹, A. P. Marscher¹, V. M. Larionov^{2,3}, J. L. Gómez⁴, H. Wiese Meyer⁵, C. Thum⁶, M. Gurwell⁷, J. Heidt⁸, and F. D. D'Arcangelo^{1,9}

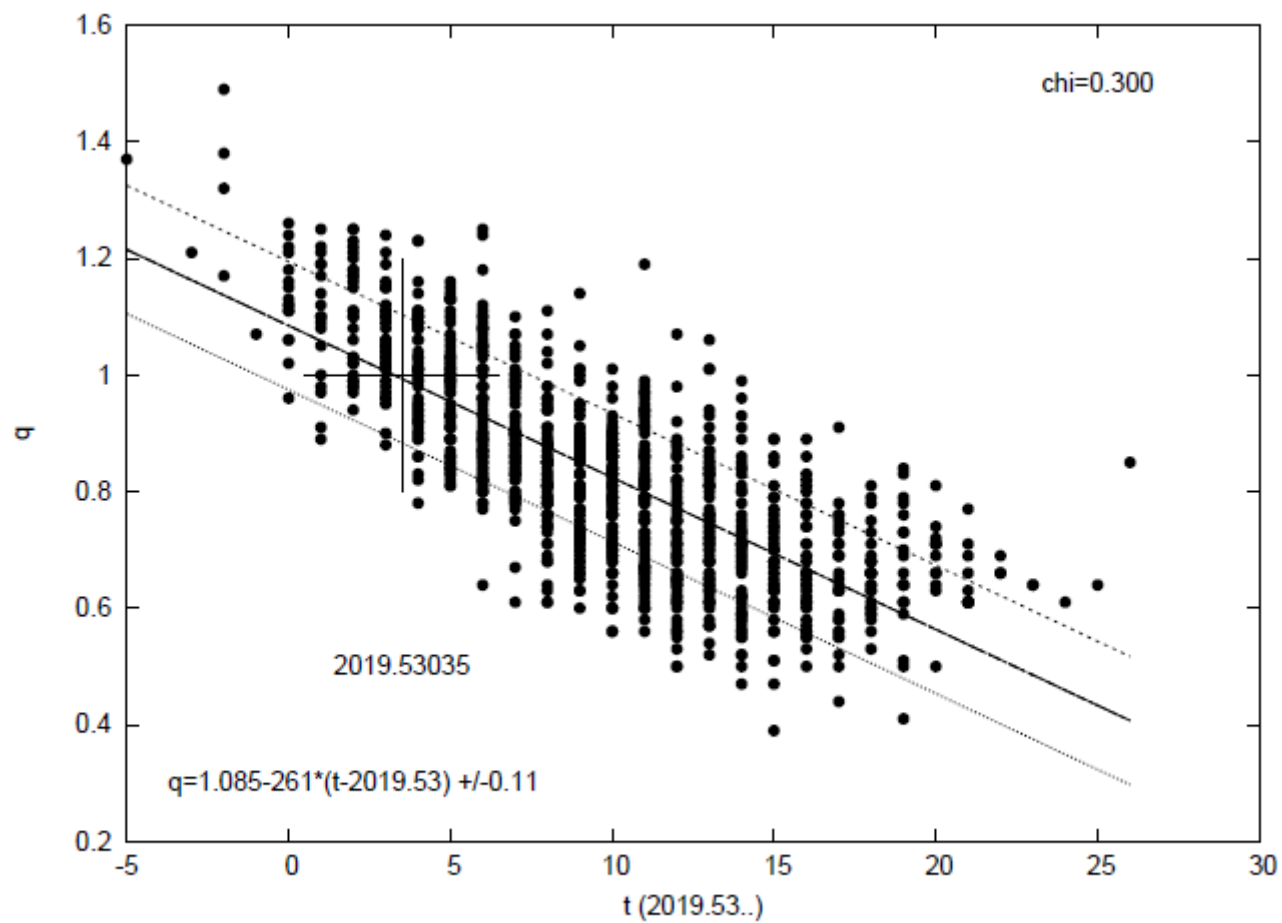




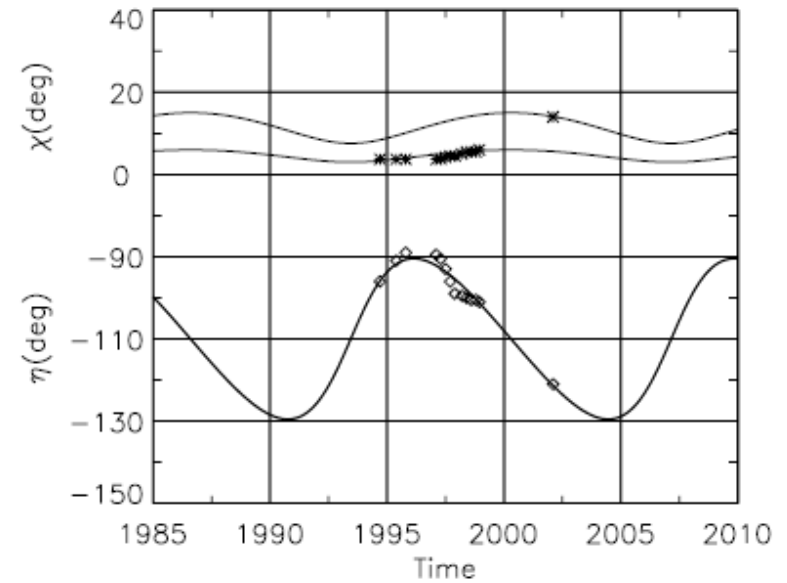
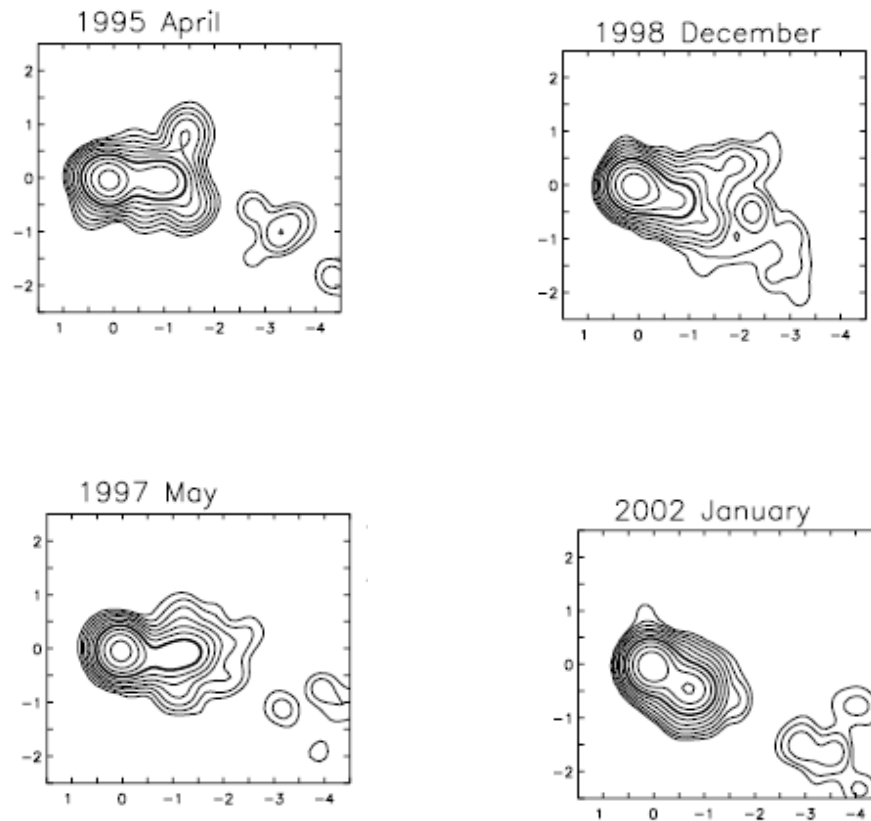
Romero, Abraham et al. 2000: Precessing jet



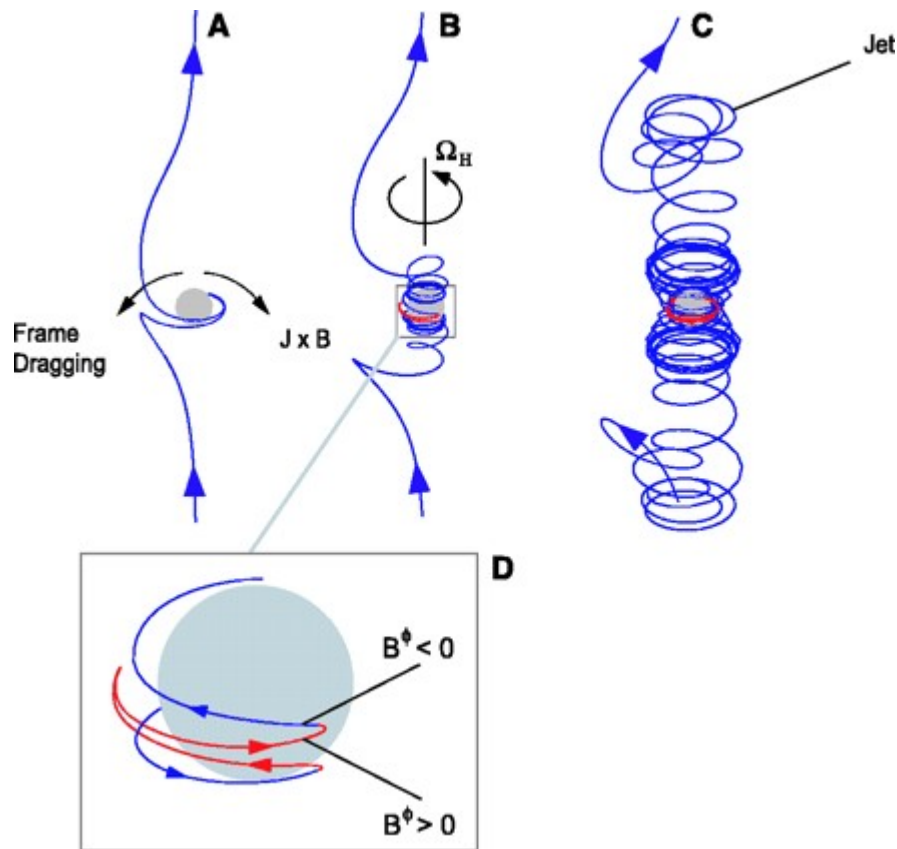


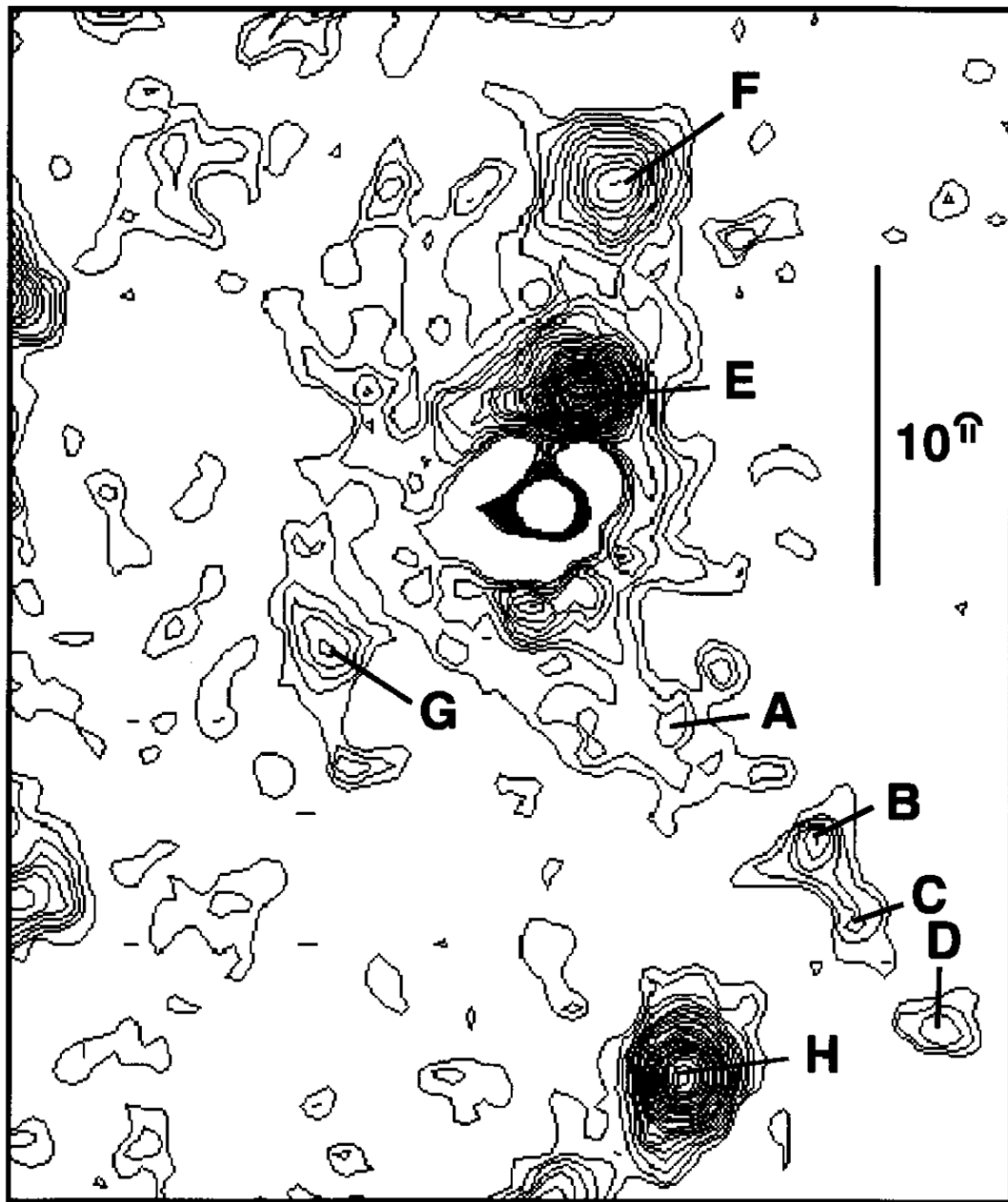


Tateyama & Kingham 2004: Precessing helical jet



Jet direction: black hole spin or accretion disk axis?





TESTING THE 1995 BINARY BLACK HOLE MODEL OF OJ287

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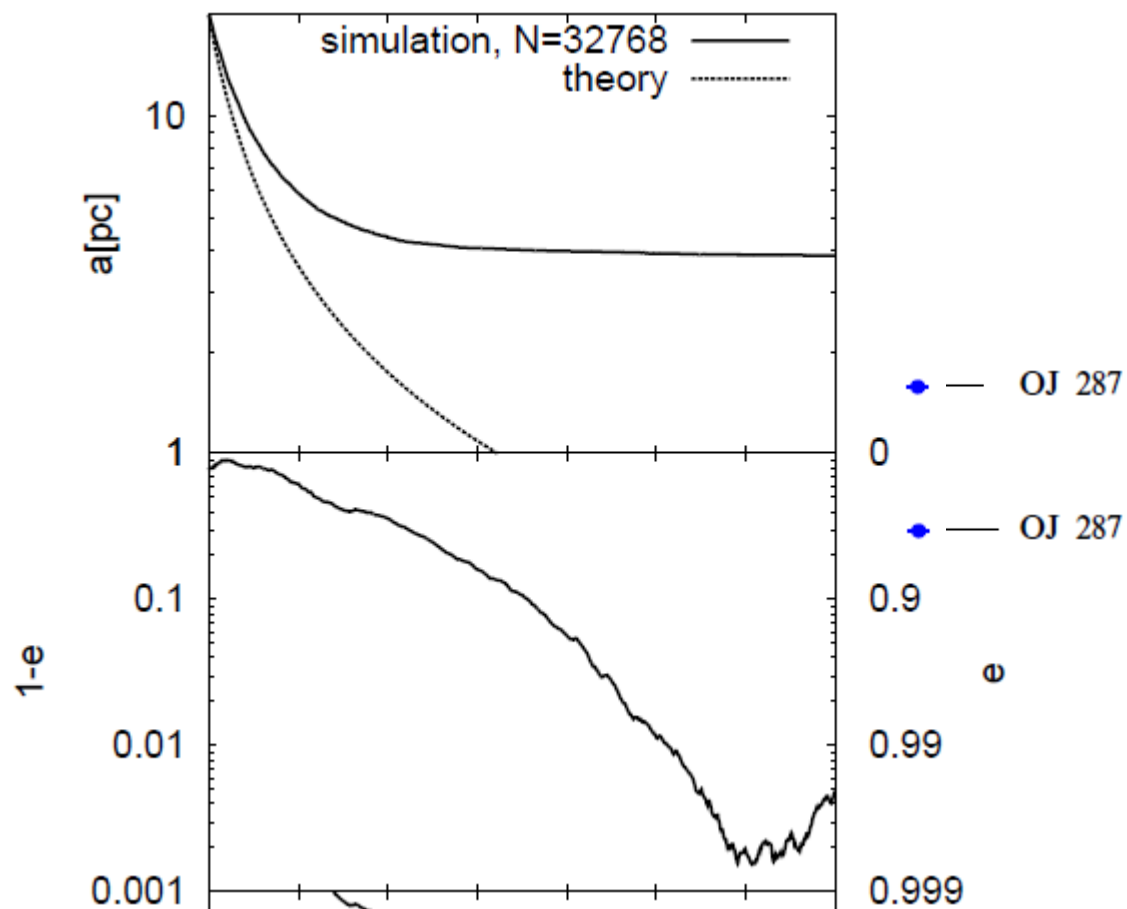
Received 2010 March 4; accepted 2010 December 31; published 2011 February 7

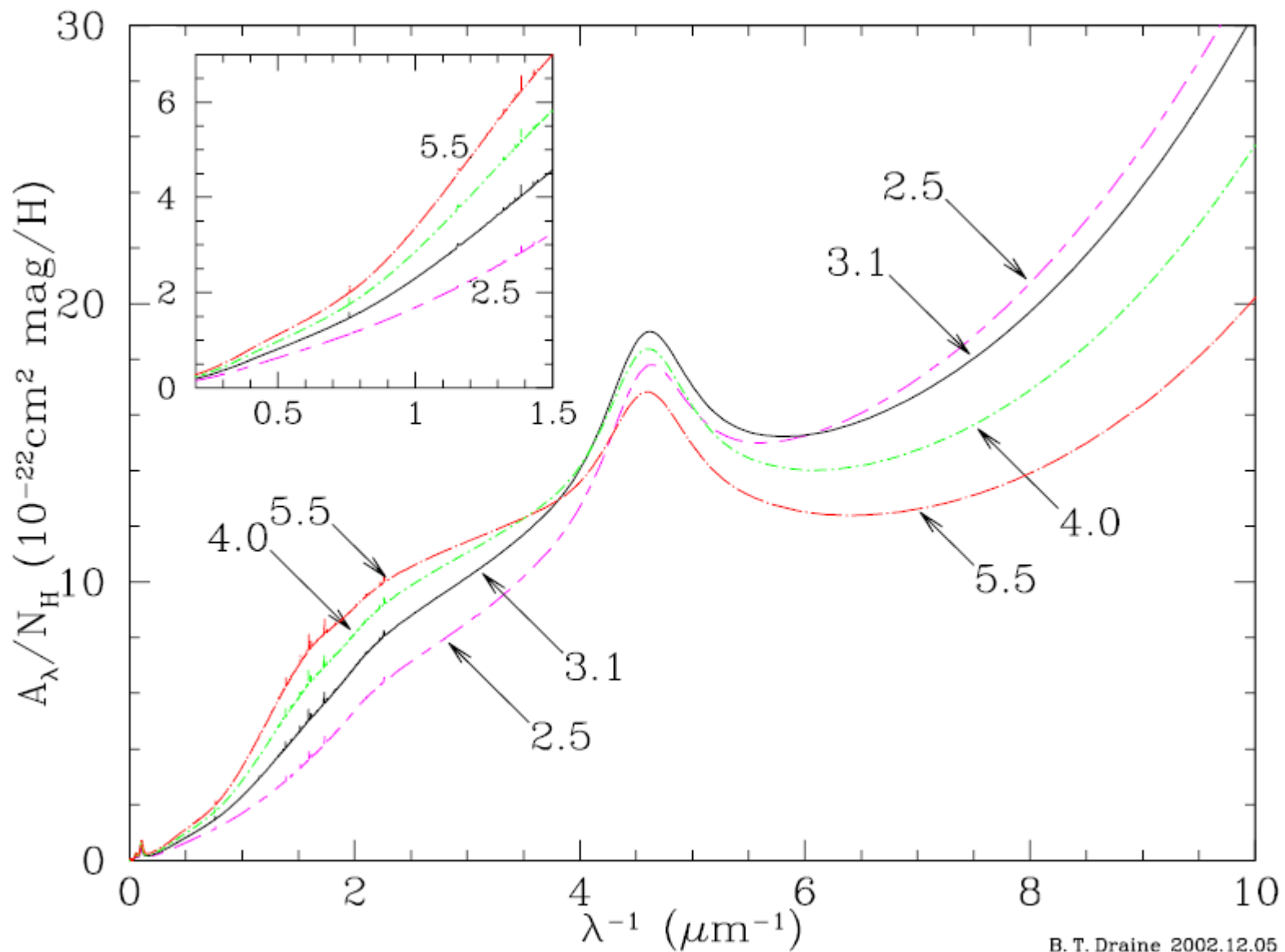
ABSTRACT

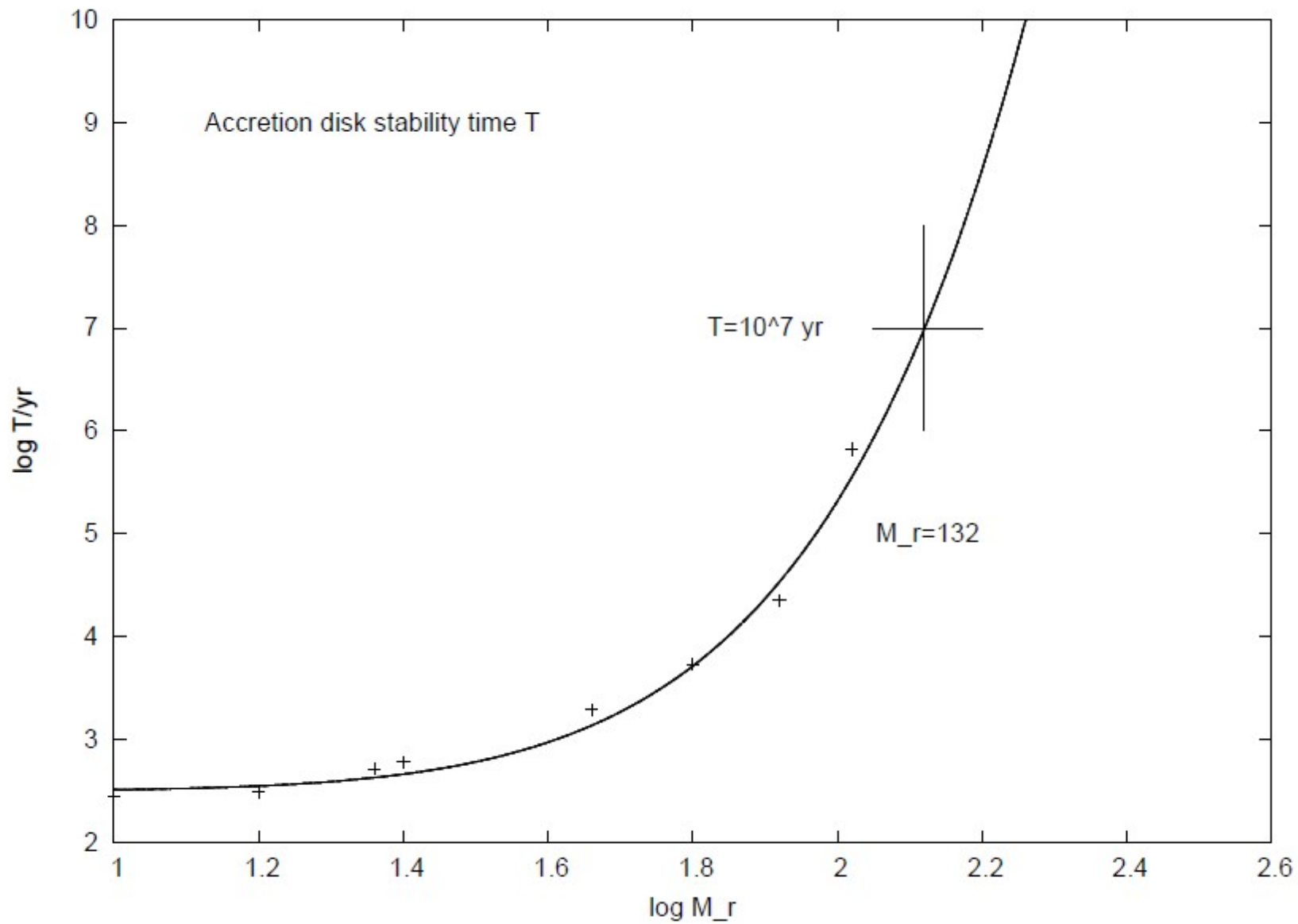
In 1995, a binary black hole model was proposed for the quasar OJ287, where the smaller secondary black hole impacts the accretion disk of the primary black hole twice during its 12 yr orbit and causes a double peak of optical outbursts. The model predicted four major outbursts and one minor outburst during the period 1996–2010. All five have now been observed. In this paper, we ask how accurate the predictions were. We use the latest optical observations from Tuorla Observatory and the KVA telescope at La Palma together with previously published data to construct a light curve for this period. We average the data in 0.04 yr bins, and subtract the observed flux from the 1995 model flux at each bin. We find that the residuals are small: they are well described by random noise of amplitude 1.4 mJy. This level is small compared with the amplitudes of the major outbursts, 5–7 mJy. Ignoring the noise, the binary model explains the optical data remarkably well.

Eccentric evolution of supermassive black hole binaries

Masaki Iwasawa^{1,2}, Sangyong An³, Tatsushi Matsubayashi⁴,
Yoko Funato⁵, Junichiro Makino²

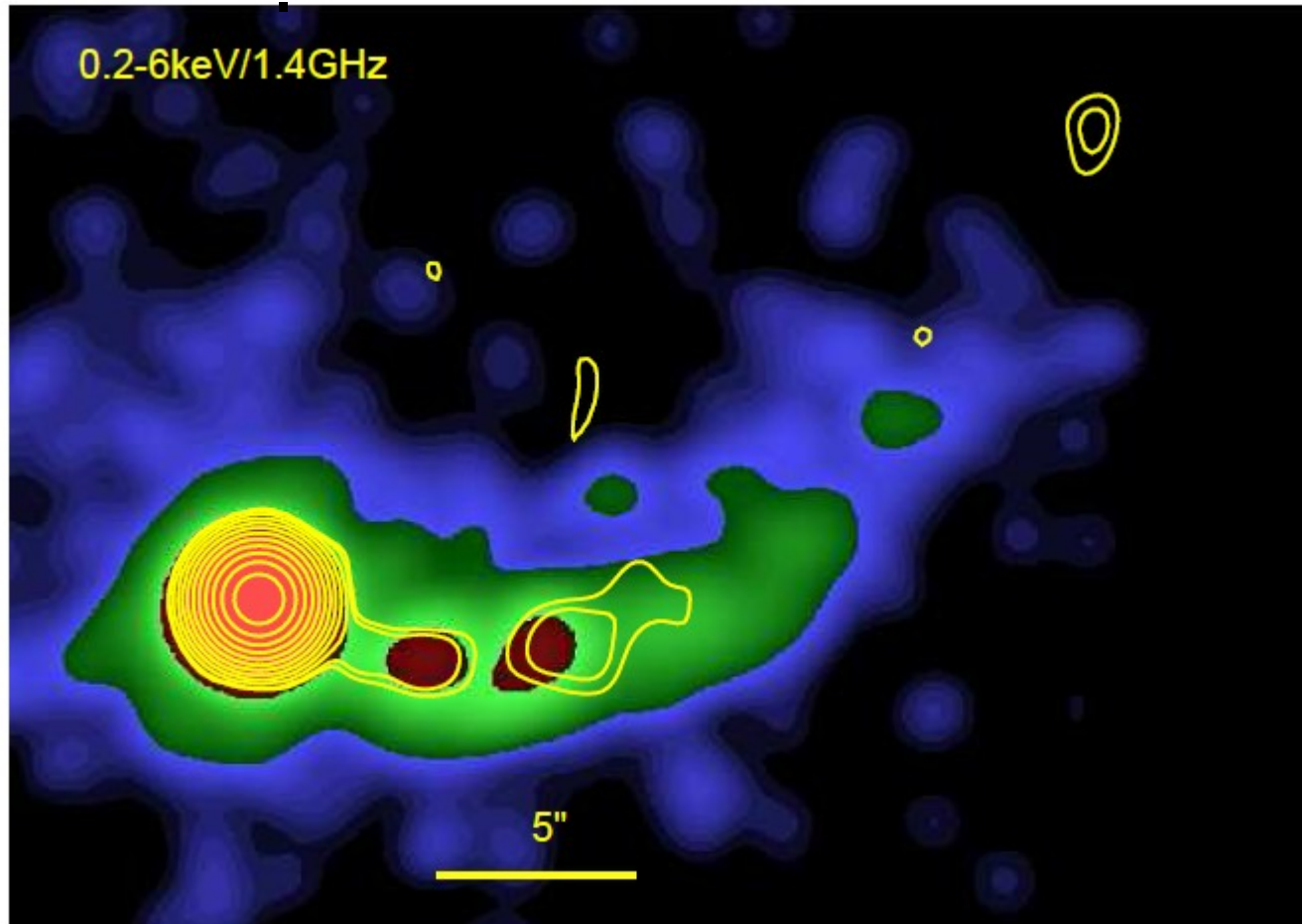




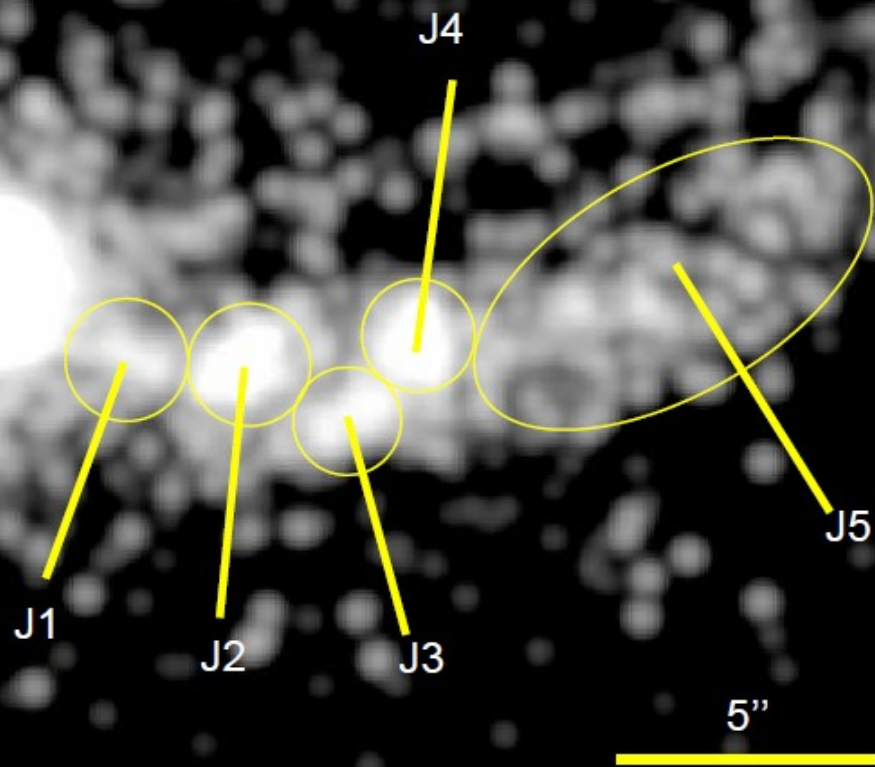


The Megaparsec-Scale X-ray Jet of the BL Lac Object OJ287

Alan P. Marscher¹ and Svetlana G. Jorstad^{1,2}



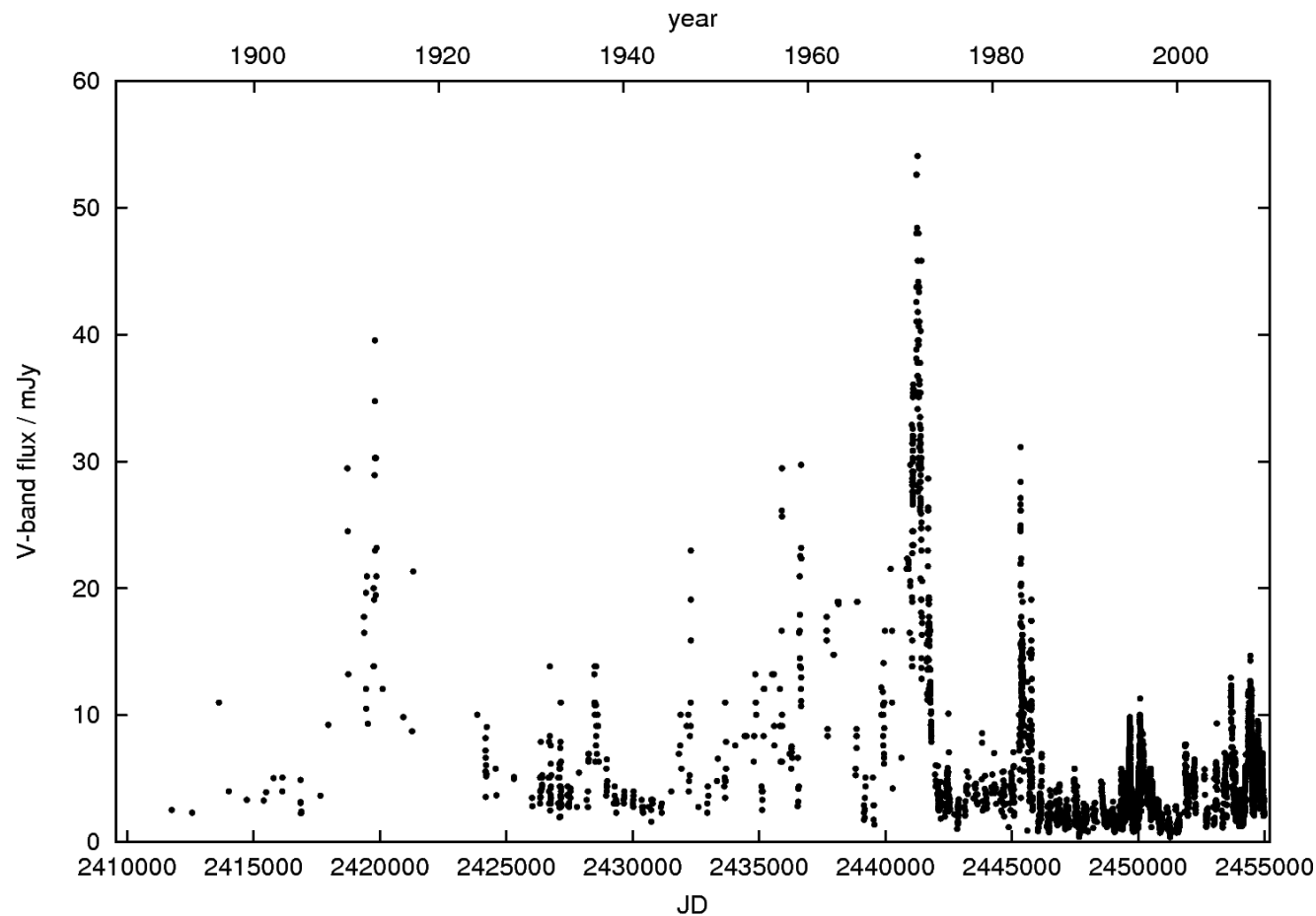
OJ287 0.2-6 keV



OJ287 light variations

Binary black hole model:

Sillanpää et al 1988



BLACK-HOLE MASS AND GROWTH RATE AT $Z \simeq 4.8$: A SHORT EPISODE OF FAST GROWTH FOLLOWED BY SHORT DUTY CYCLE ACTIVITY

BENNY TRAKHTENBROT¹, HAGAI NETZER¹, PAULINA LIRA² AND OHAD SHEMMER³

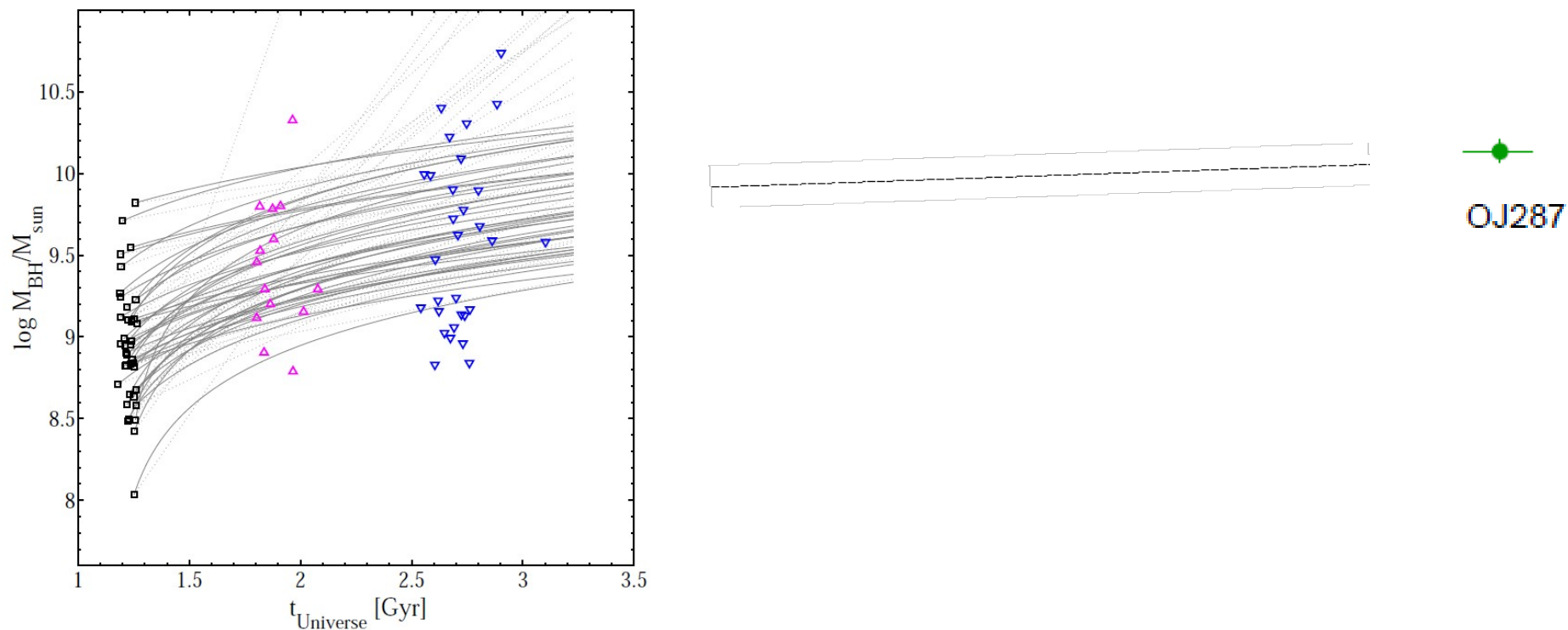


Figure 8. Evolution scenarios for the $z \simeq 4.8$ SMBHs with various duty cycles. Symbols are identical to those in Fig. 4. Solid lines describe M_{BH} growth under the assumption of constant L_{bol} and a duty cycle of 20%. Dotted lines represent the constant L/L_{Edd} scenario and a duty cycle of 10%.

Jet-disk connection in OJ287

Mauri J. Valtonen^{1,2}, Tuomas Savolainen³ and Kaj Wiik²

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²Dept. of Physics & Astronomy, Tuorla Observatory, University of Turku, 21500 Piikkiö,
Finland
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³Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany
email: tsavolainen@mpifr-bonn.mpg.de

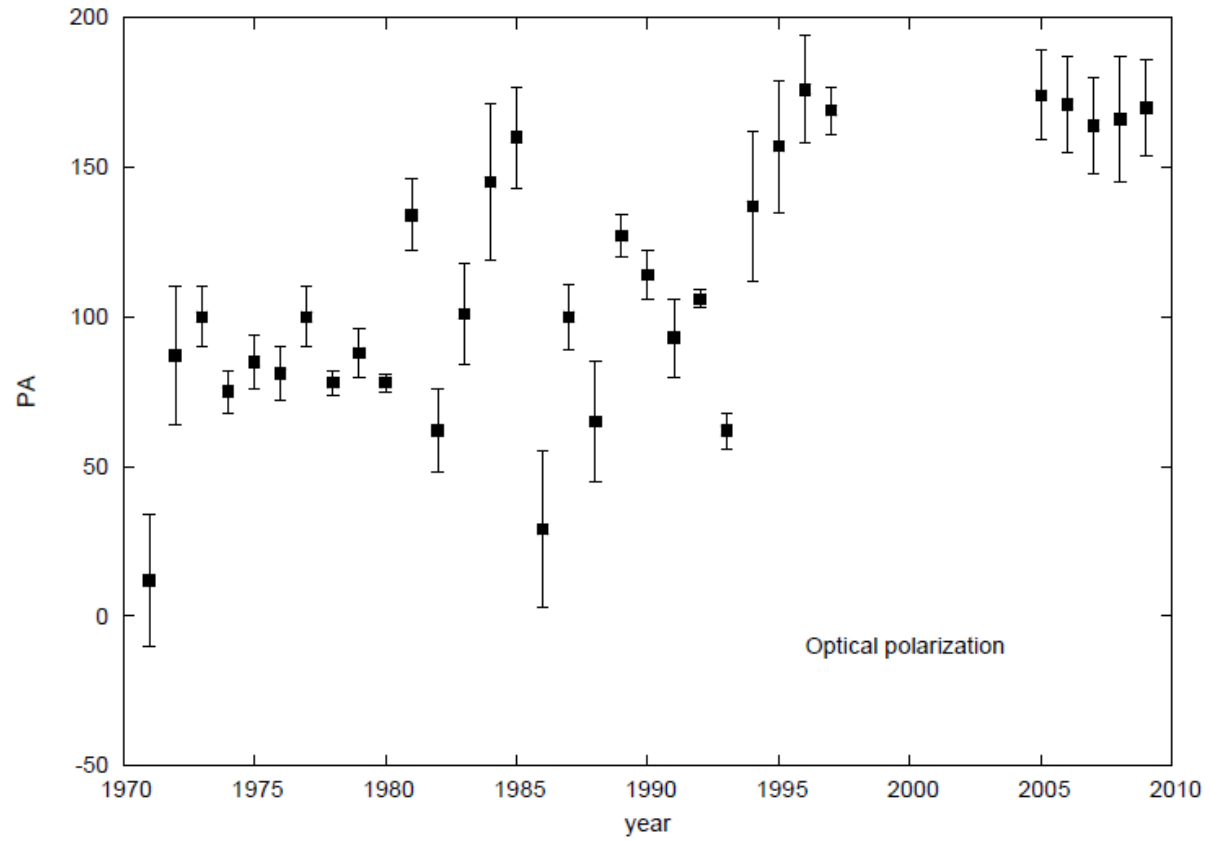
Optical polarization angle in OJ287 and the binary black hole model

Mauri J. Valtonen¹ and Carolin Villforth^{2,3*}

¹*Helsinki Institute of Physics, FIN-00014 University of Helsinki, Finland*

²*Tuorla Observatory, Department of Physics and Astronomy, University of Turku, Väisäläntie 20, FI-21500 Piikkiö, Finland*

³*Space Telescope Science Institute, 3700 San Martin Drive, 21218 Baltimore, Maryland, USA*



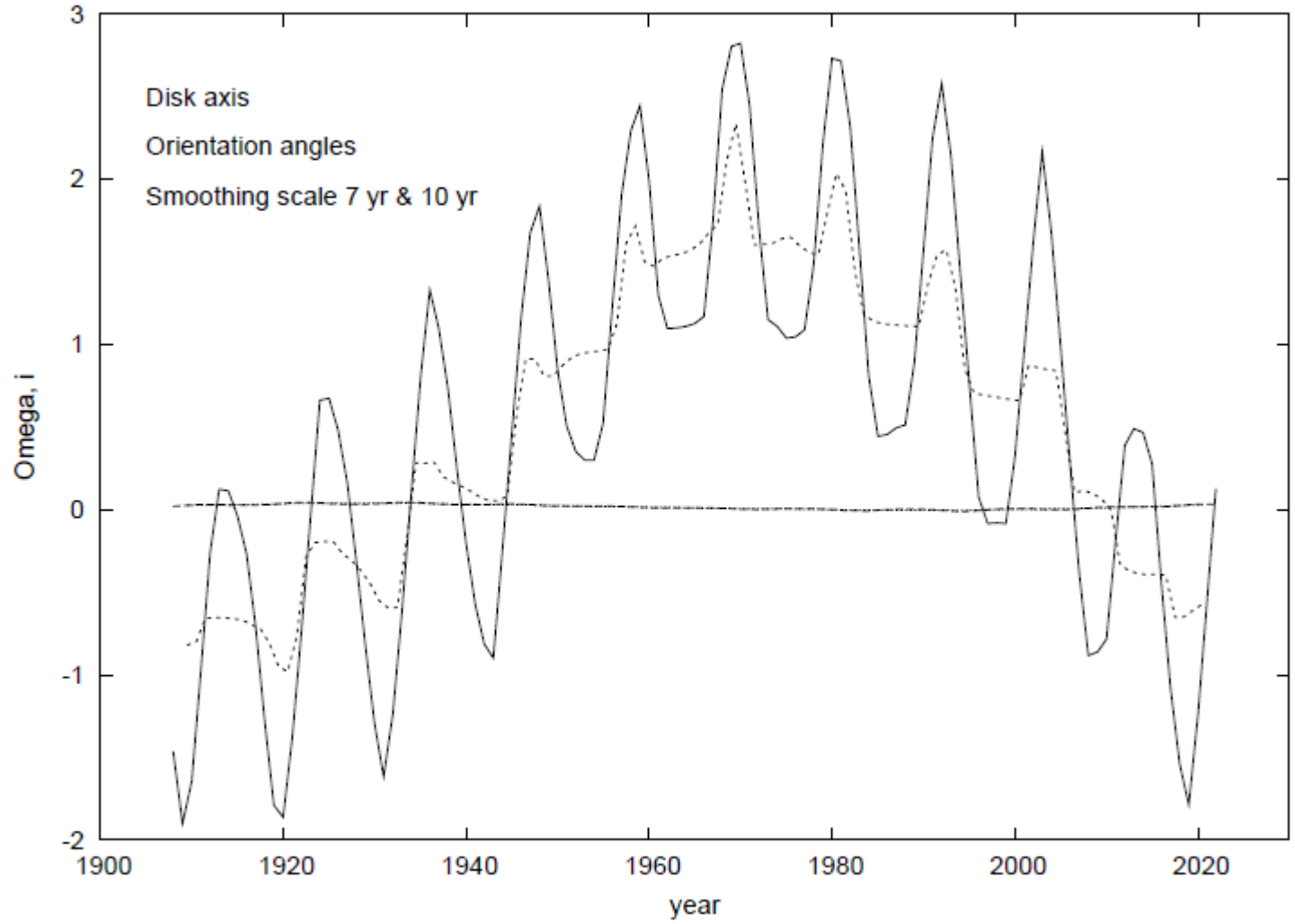


Figure 2. The evolution of the mean disk in the binary model.

Kozai cycle

$$\frac{di}{d\tau} = 0 \tag{1}$$

$$\frac{d\Omega}{d\tau} = -\frac{3}{4}\cos i \tag{2}$$

(Valtonen and Karttunen 2006) where τ is the normalized time coordinate,

$$\tau \sim \frac{P}{P_e} \frac{t}{P} \tag{3}$$

and t is time, and P and P_e are the inner and outer periods,

$$P_e \sim 12 \text{ yr}, \quad P \sim P_e/5, \quad \text{and} \quad P_{Kozai} \sim 120 \text{ yr}$$

Jet perpendicular to disk: optical base flux determines viewing angle, Gamma

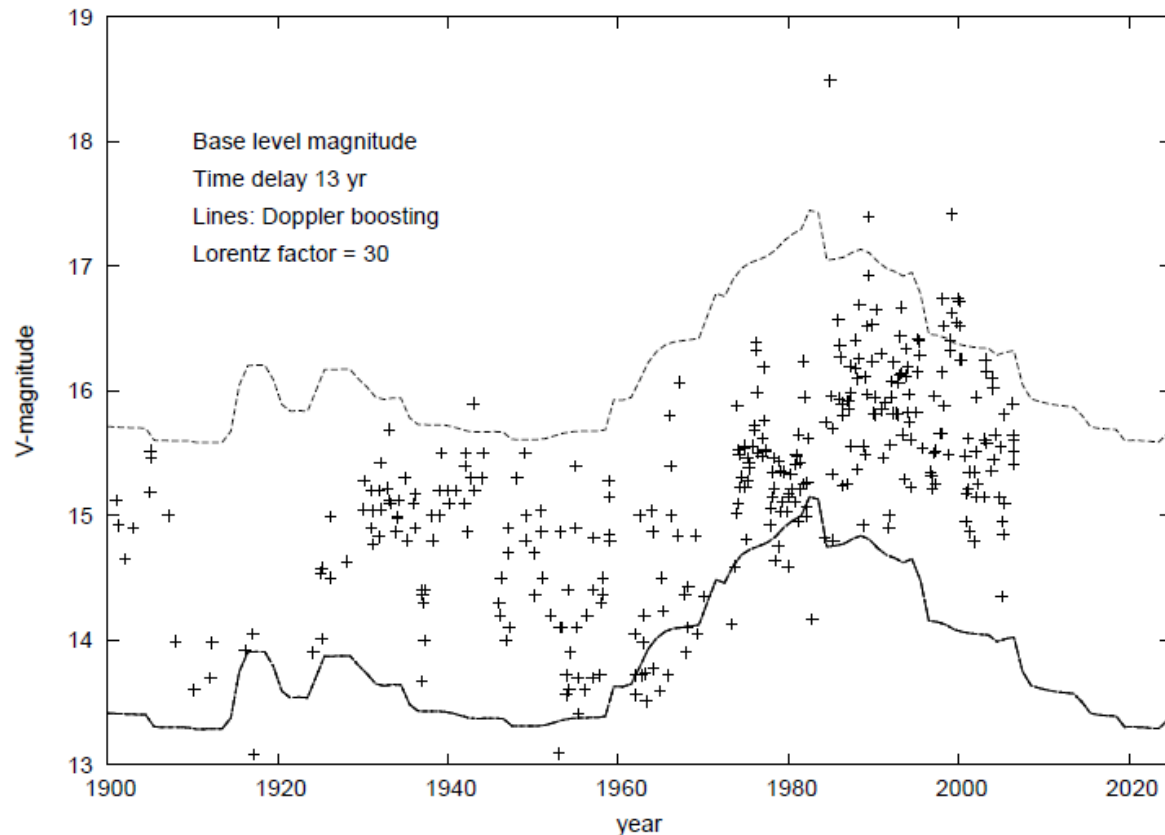
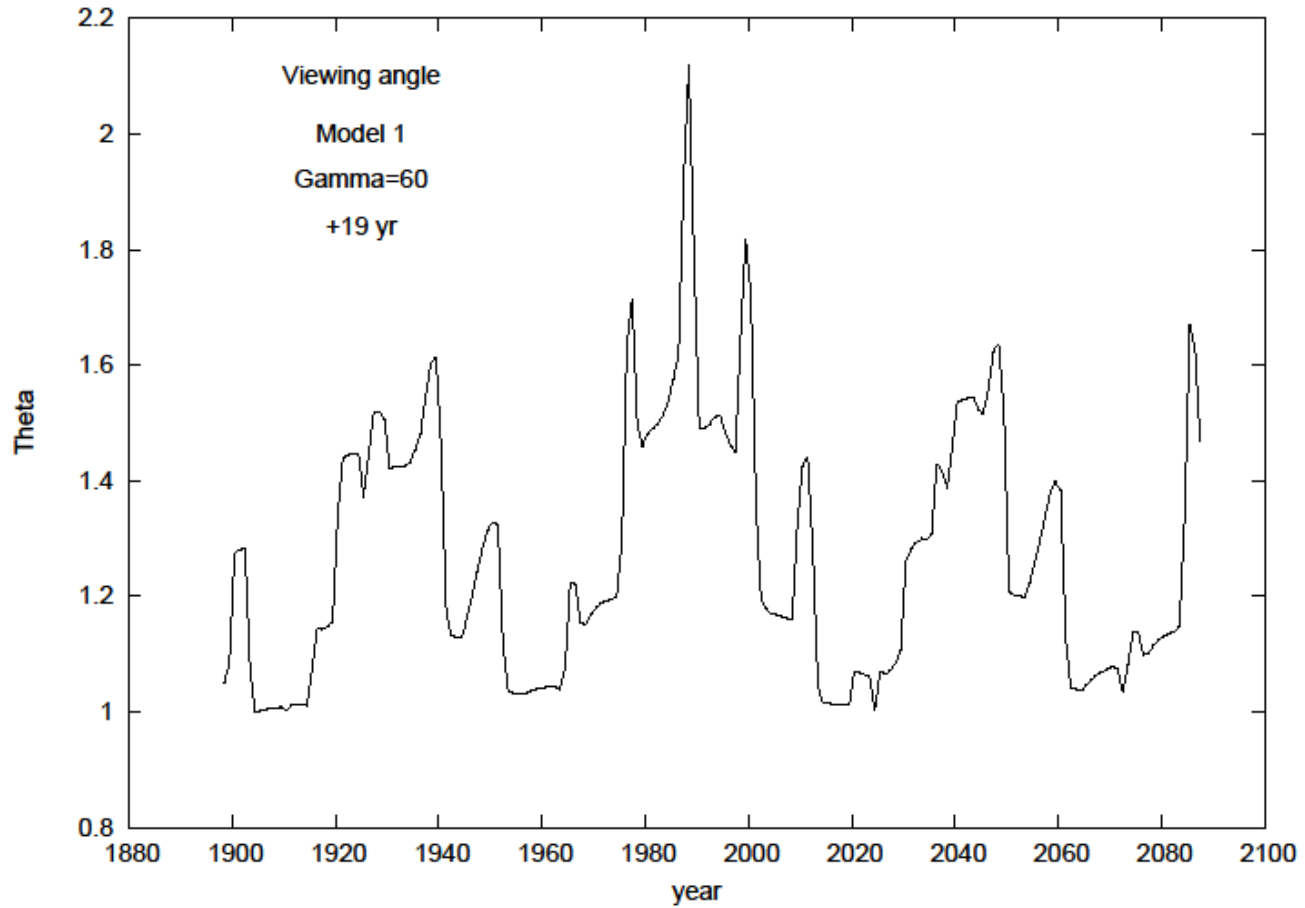


Figure 3. The variation of the base level brightness in OJ287.

Evolution of viewing angle



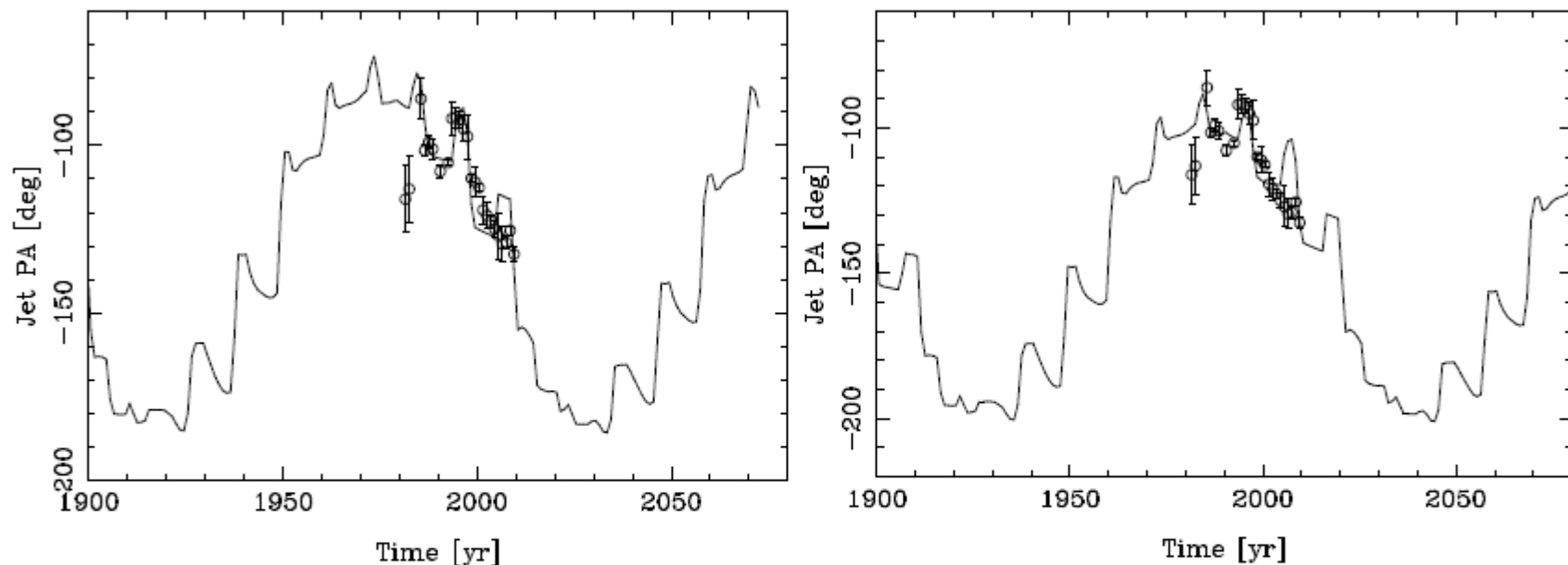


Figure 1. The line shows the variation of the jet angle as a function of time, based on the binary model of OJ287. The viewing angle of the jet varies between 1 and 2 degrees. The circles represent the yearly average PA observed in the 2-6 cm wavelength range. The error bars represent the standard deviation of the PA when more than two observations per year were available. *Left:* A 4-year lag is introduced between changes in the accretion disk orientation and jet reorientation. *Right:* The same as the left-hand side, but the lag is 14 years.

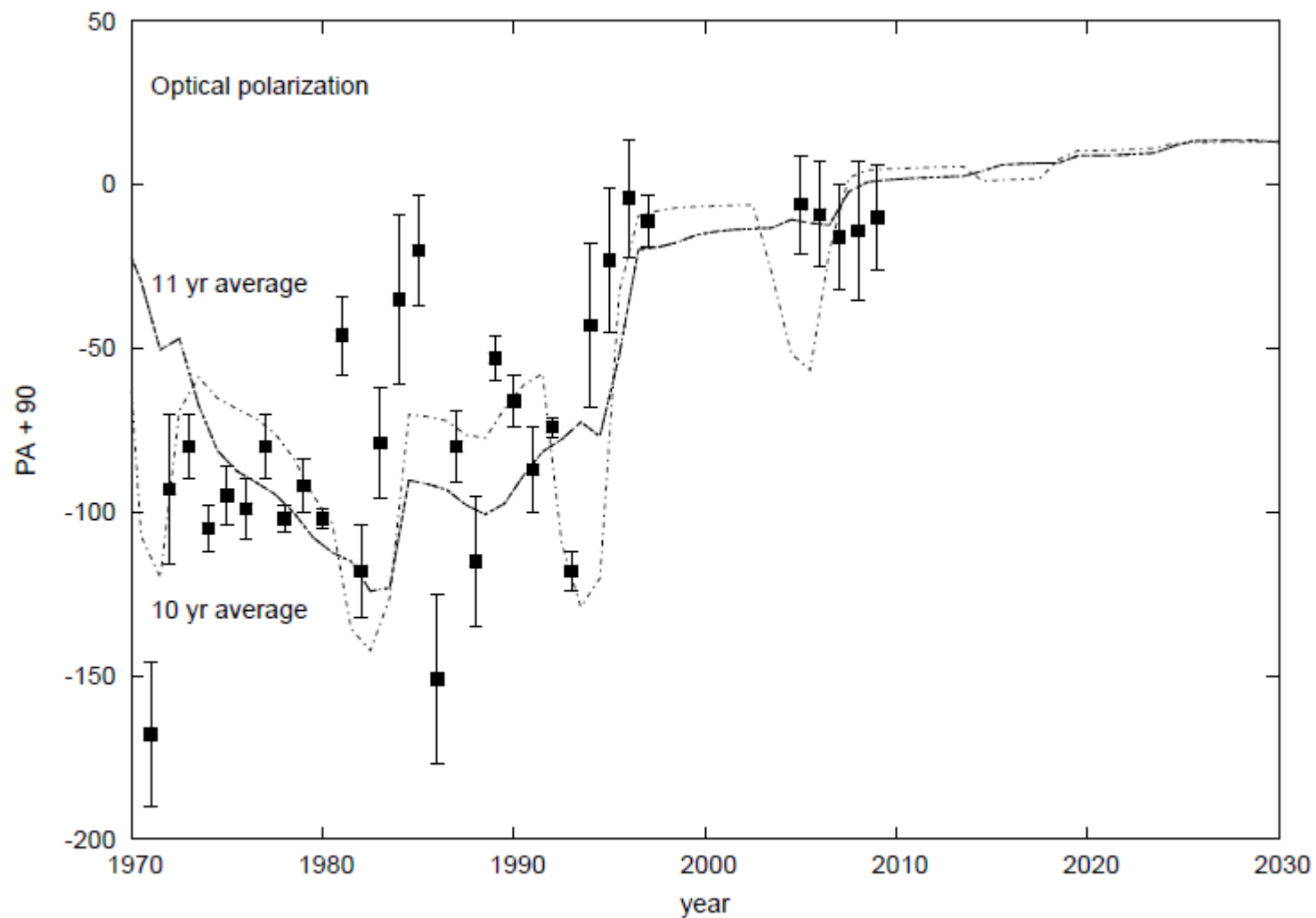
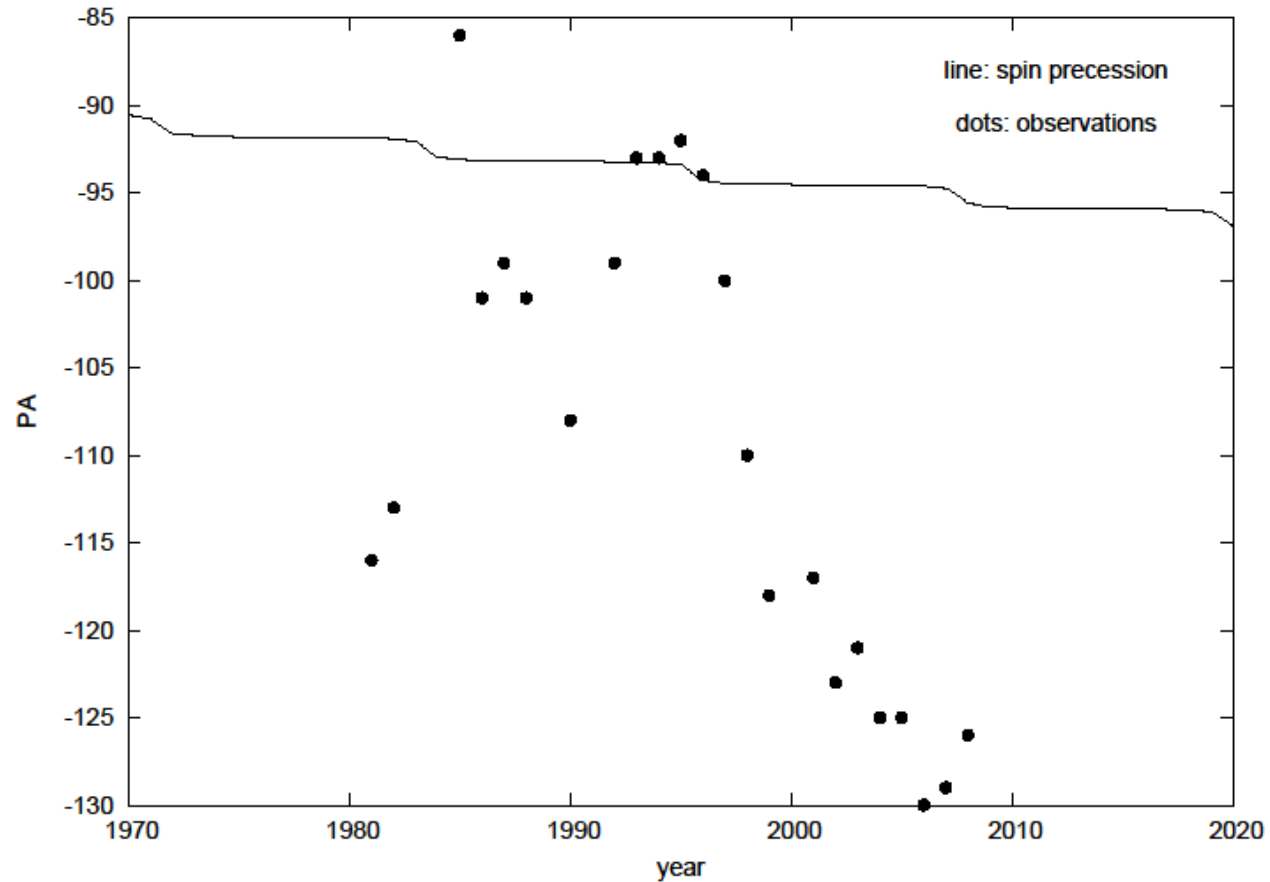


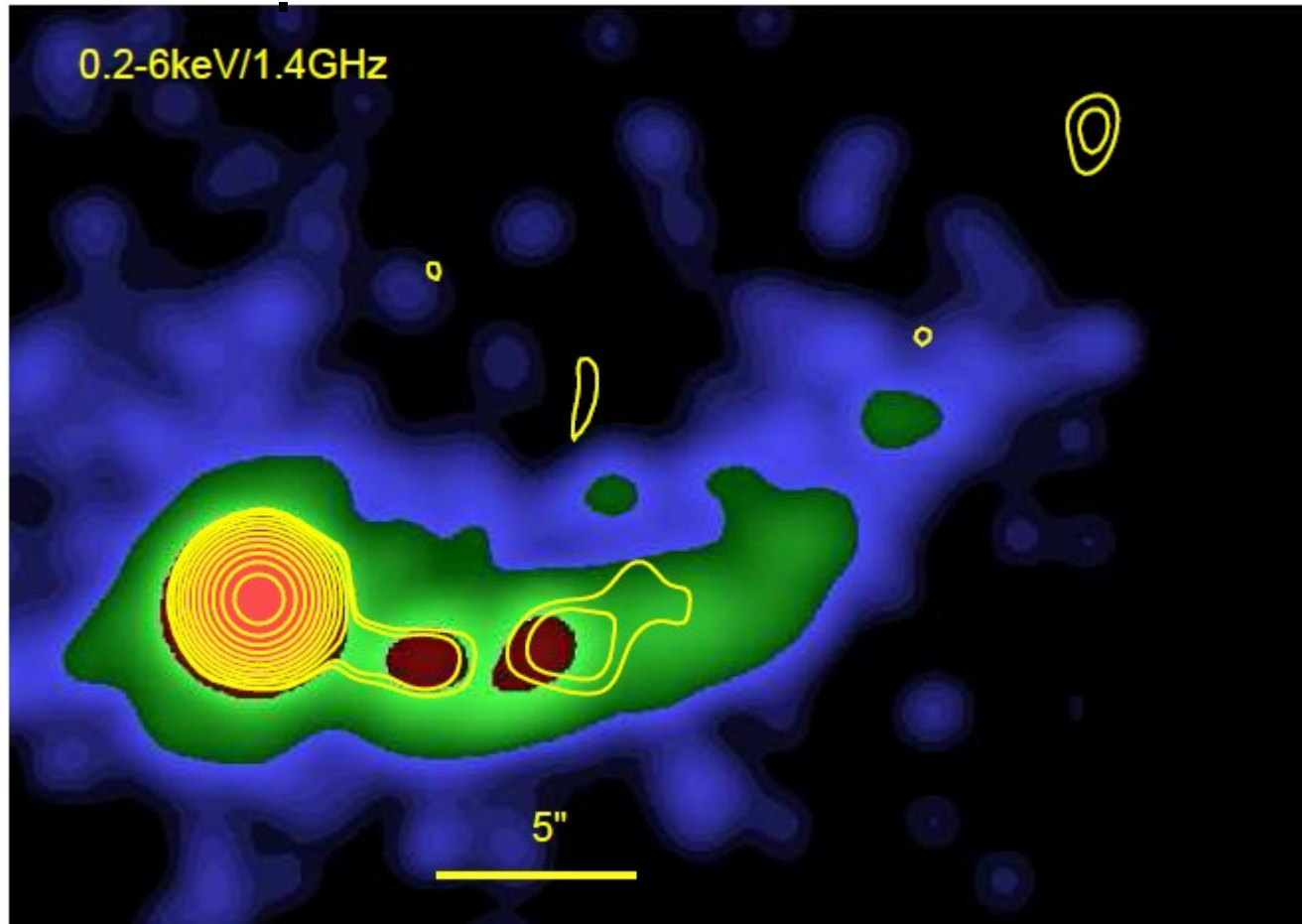
Figure 5. The evolution of the optical polarization angle com-

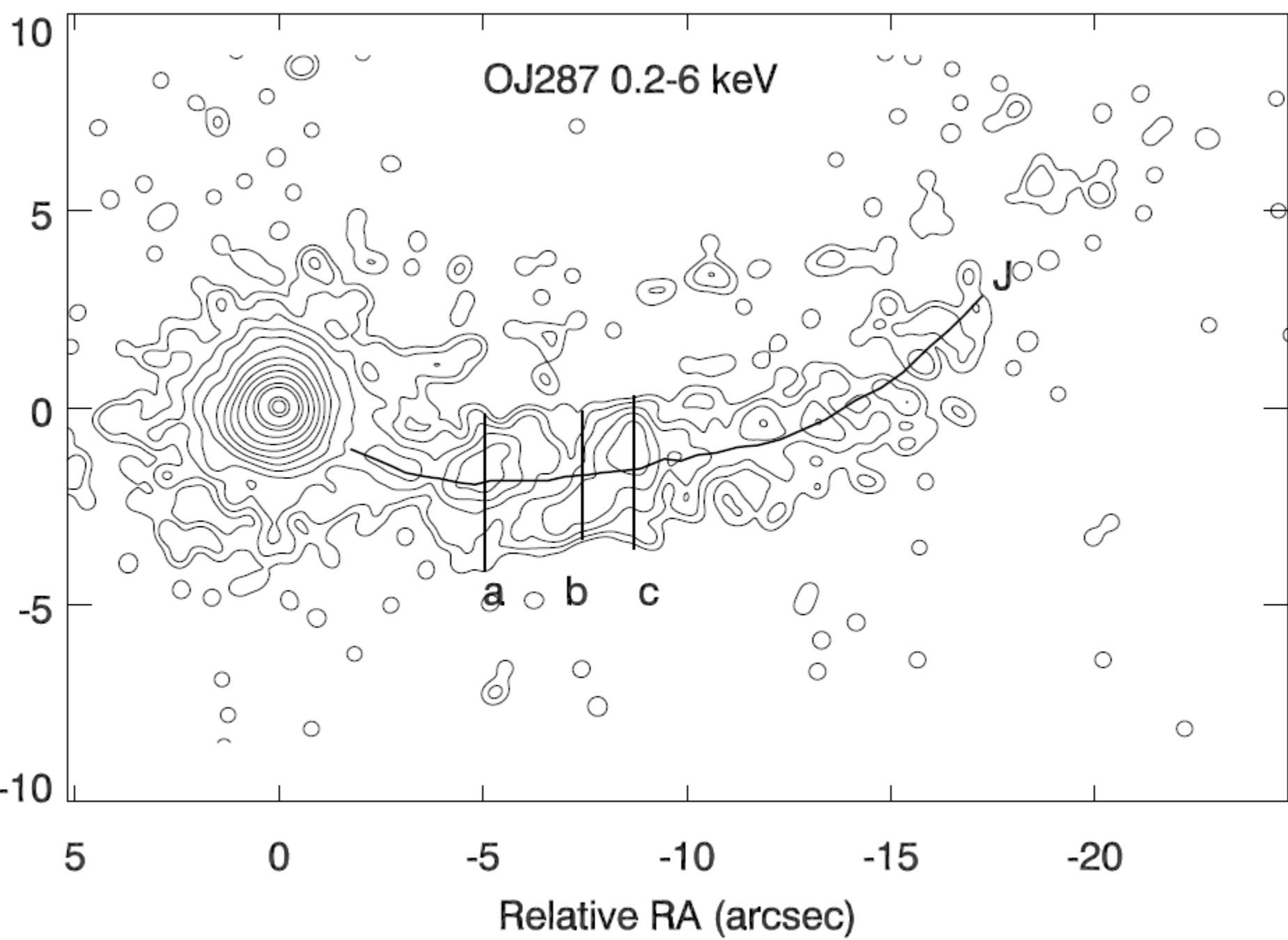
Projected spin angle vs. observations



The Megaparsec-Scale X-ray Jet of the BL Lac Object OJ287

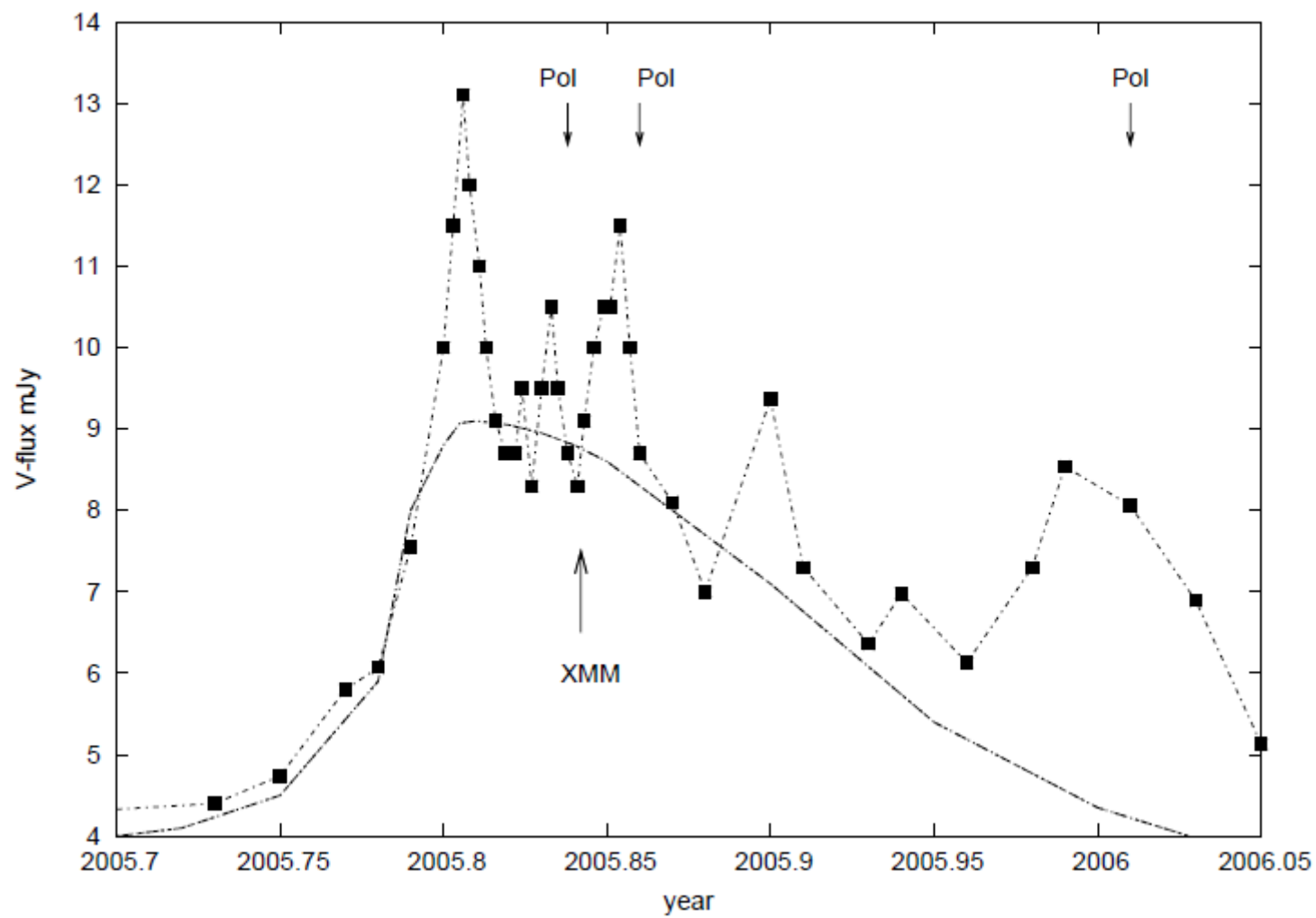
Alan P. Marscher¹ and Svetlana G. Jorstad^{1,2}





Conclusions

- Small scale jet direction given by the disk axis, not BH spin axis
- Spin rotation shows up in the wiggles of the large scale jet
- Overall jet curvature may be related black hole binary history
- There is a delay in reorientation between radio and optical jet regions
- Radio jet should stop rotation in the sky and start reversing at about 2030



$$t_d = 0.75$$

implies

$$\alpha_g \text{ r } \dot{m}/\dot{m}_{\text{edd}} = 14$$

$$\text{If } \dot{m}/\dot{m}_{\text{edd}} = 0.01$$

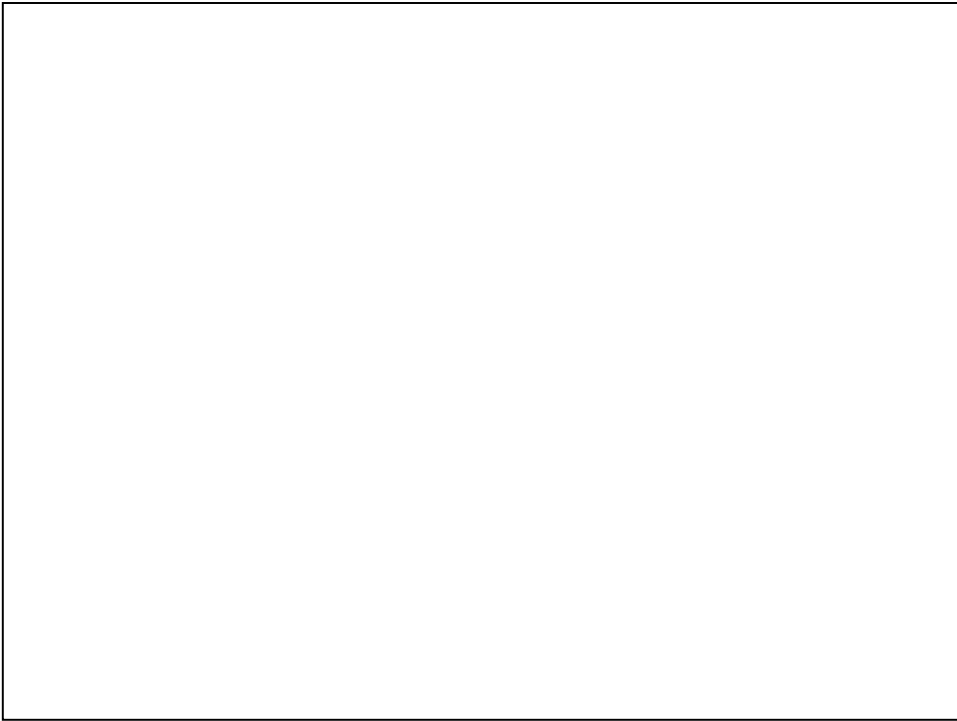
$$\alpha_g = 0.14$$

The 1995 model vs.

Observations . A chi-square test (with 234 degrees of freedom) gives $\chi^2 = 205$, which corresponds to the probability $P = 0.9$ that the two distributions agree with each other. The corresponding correlation coefficient is 0.58.

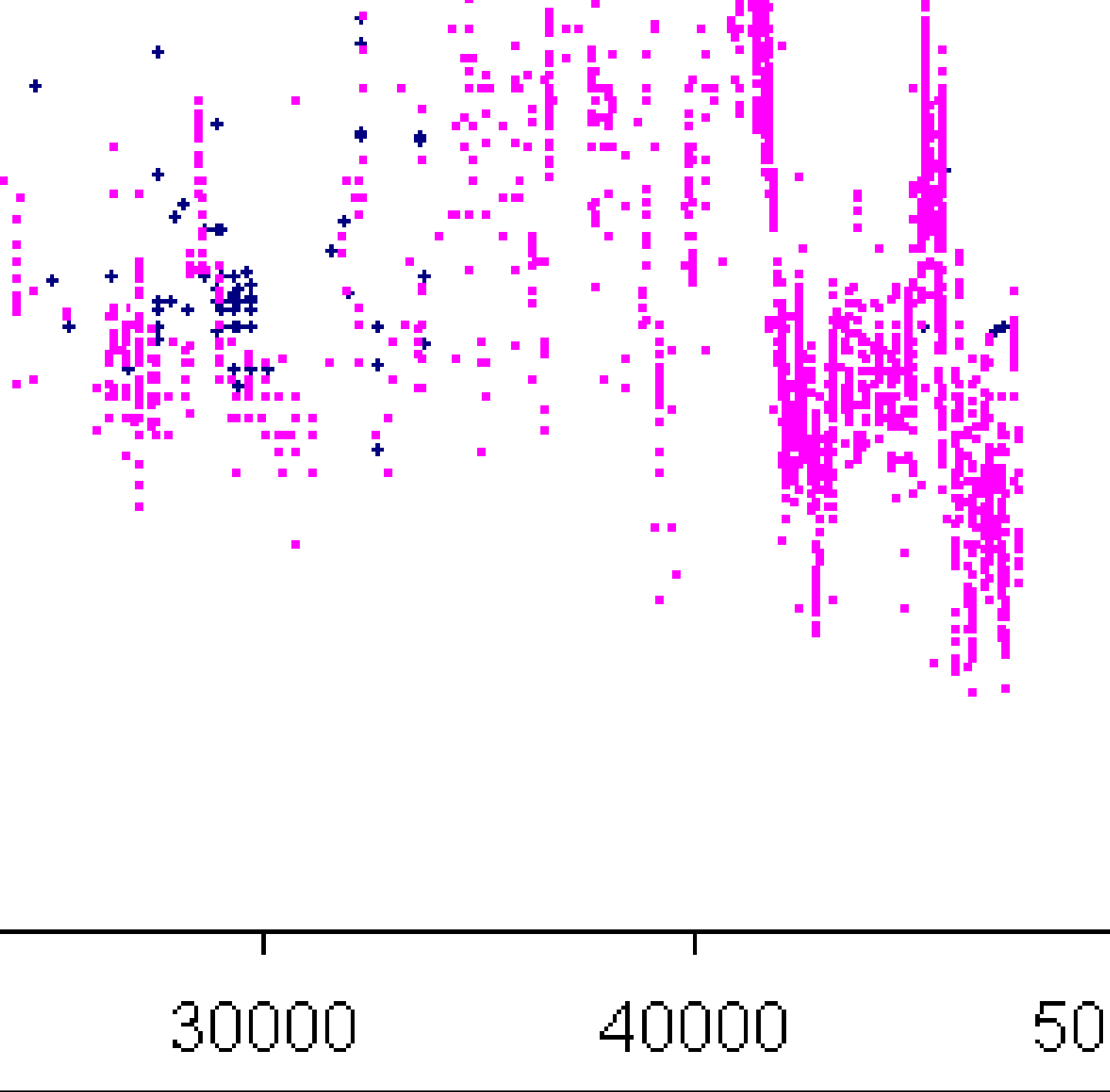
CORRELATIONS: BEST OF 10^6 TRIALS

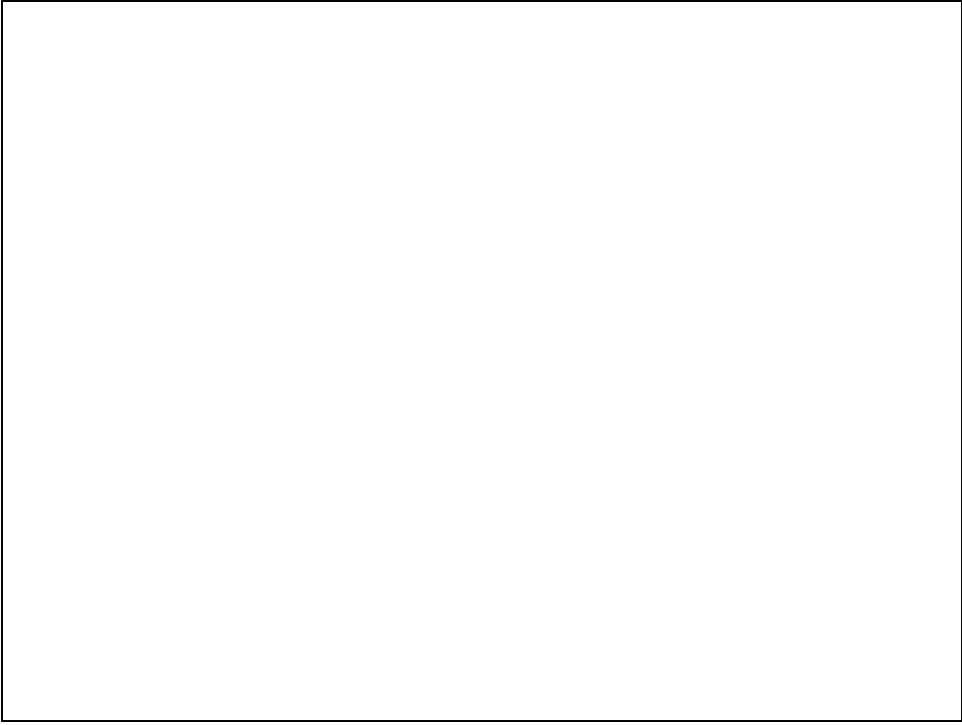
Correlated Quantities	χ^2	$P(\chi^2)$	corr coeff
Obs vs. $\beta = 1$ model	310	6×10^{-4}	0.30
Obs vs. $\beta = 2$ model	305	10^{-3}	0.32
Obs vs. QPO model	290	7×10^{-3}	0.37
Obs vs. PO model	205	0.9	0.58



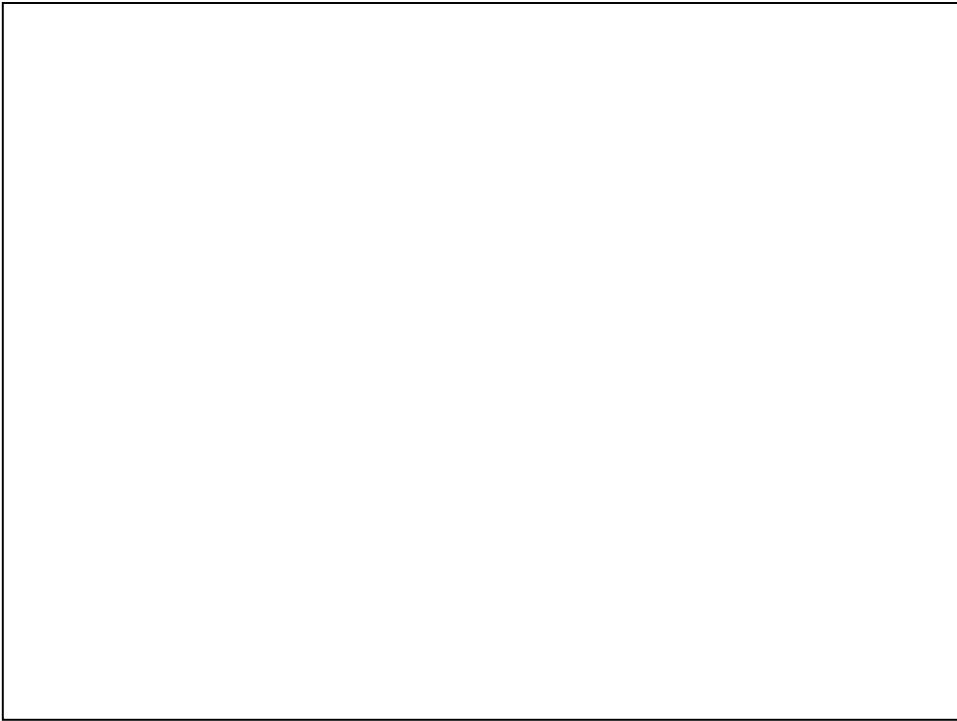








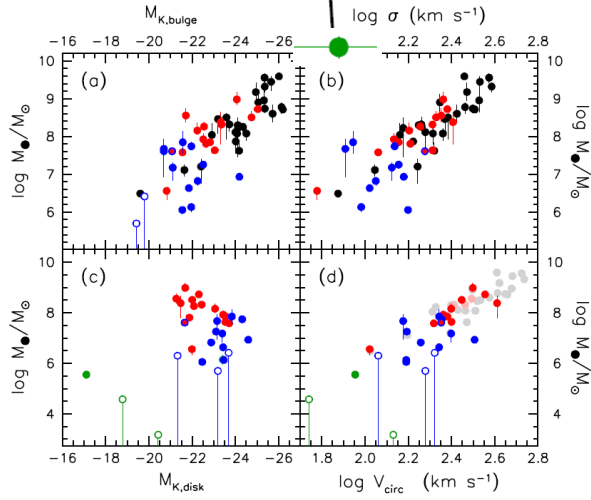




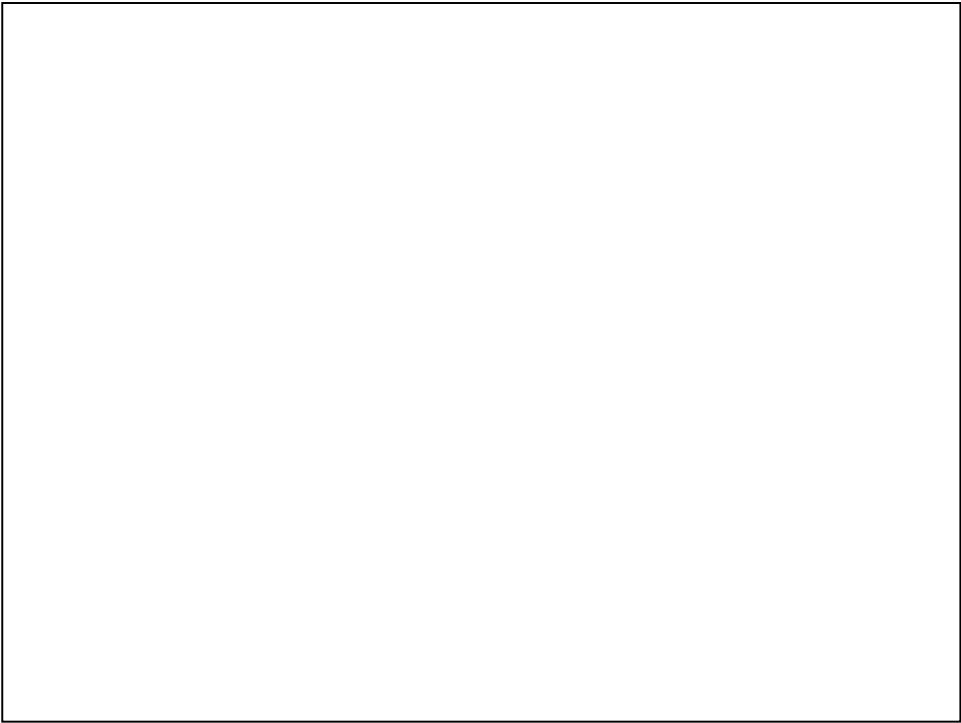
Supermassive black holes do not correlate with dark matter halos of galaxies

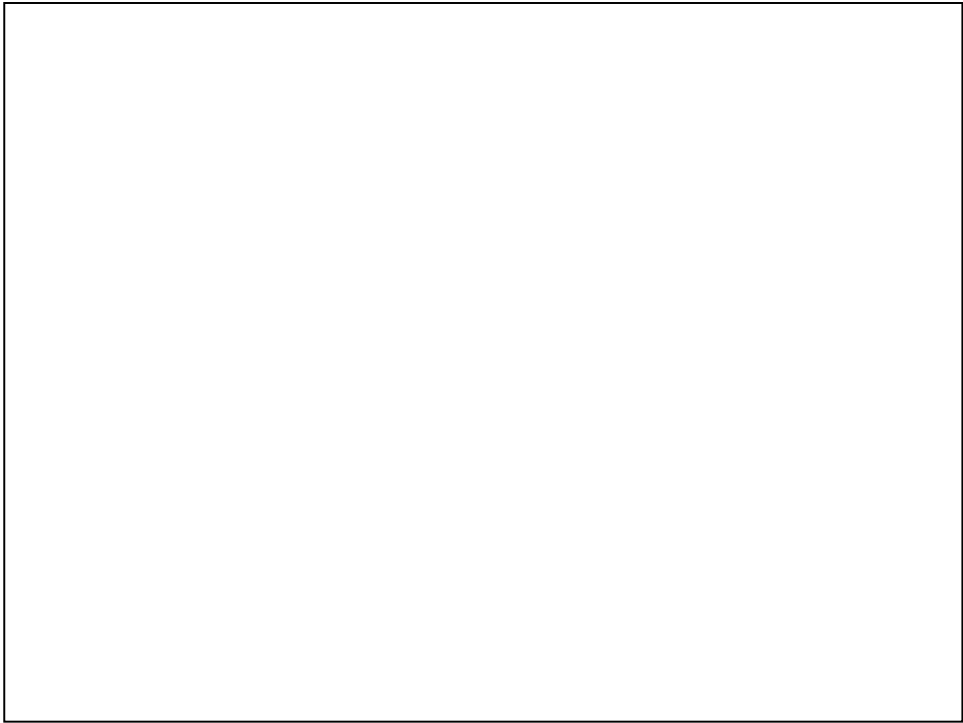
John Kormendy^{1,2,3} & Ralf Bender^{2,3}

OJ 287

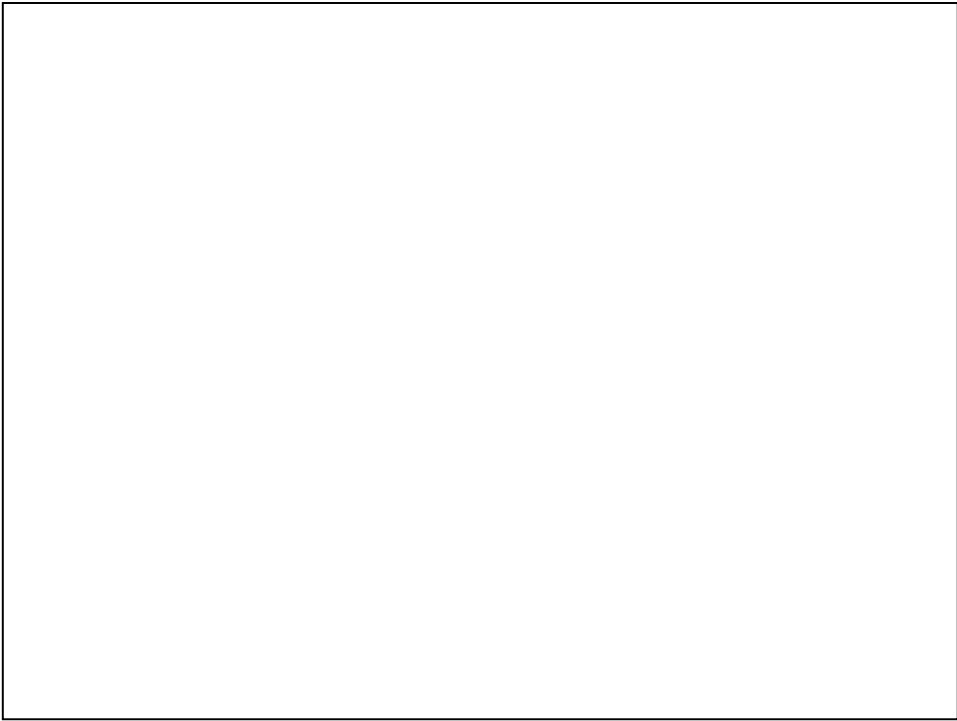


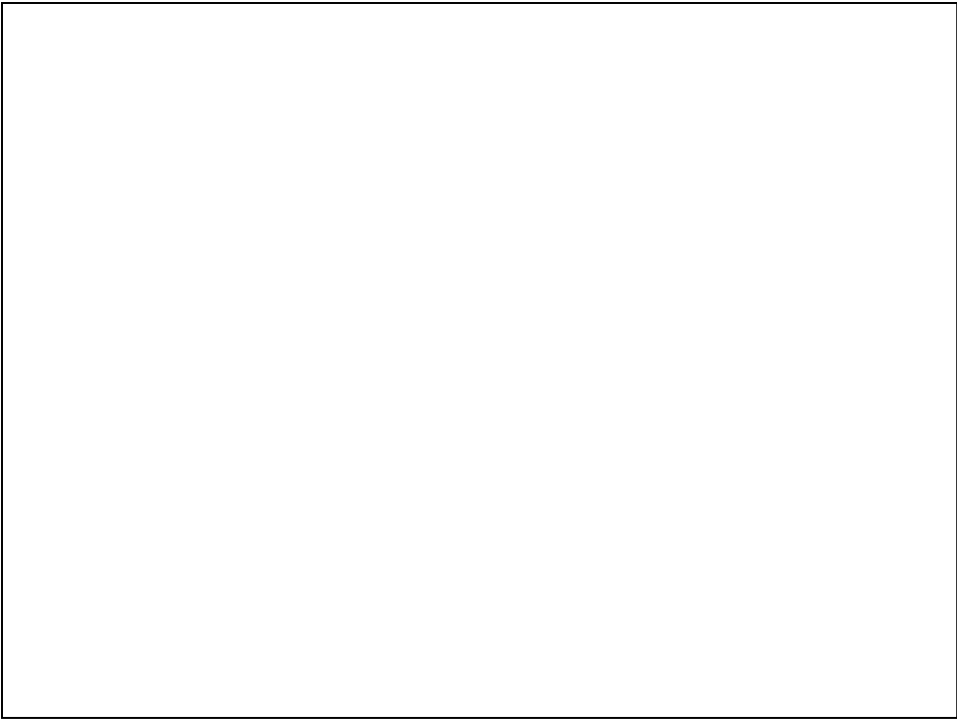




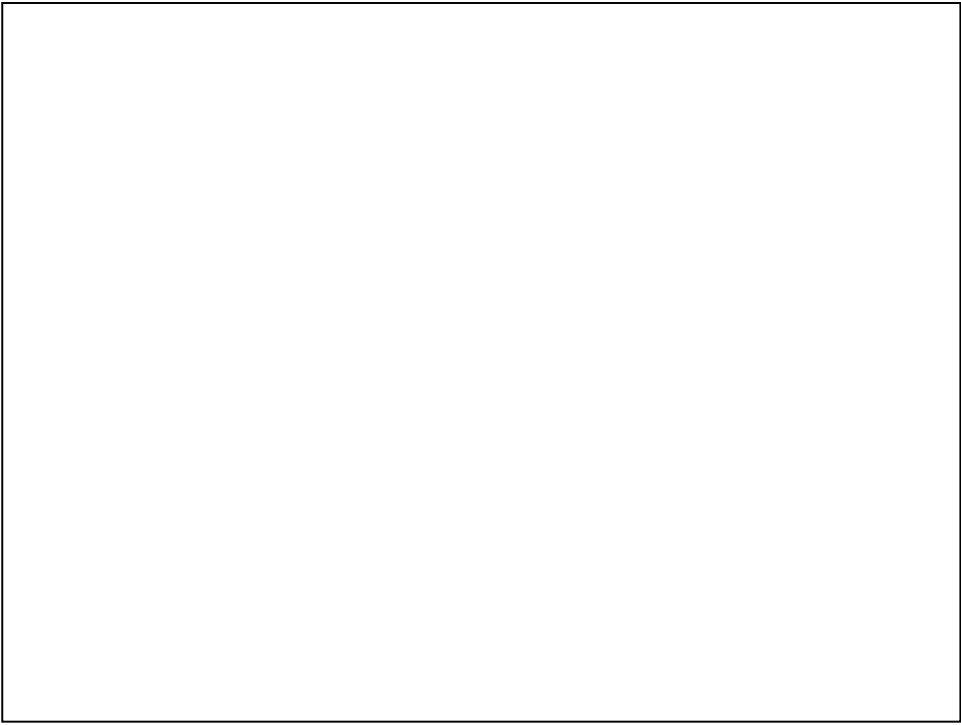




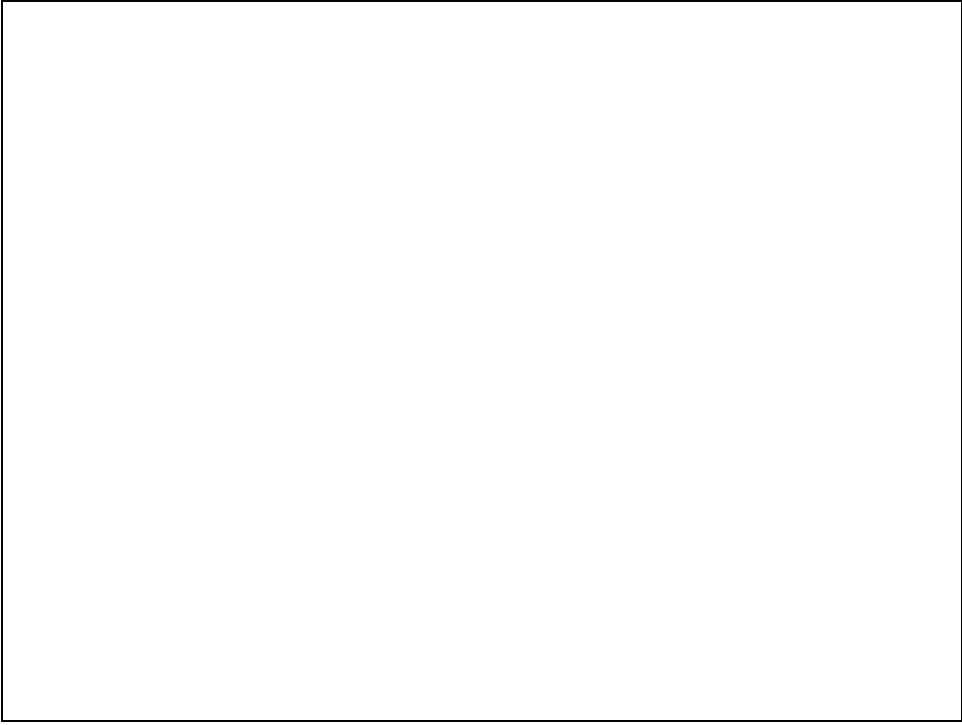


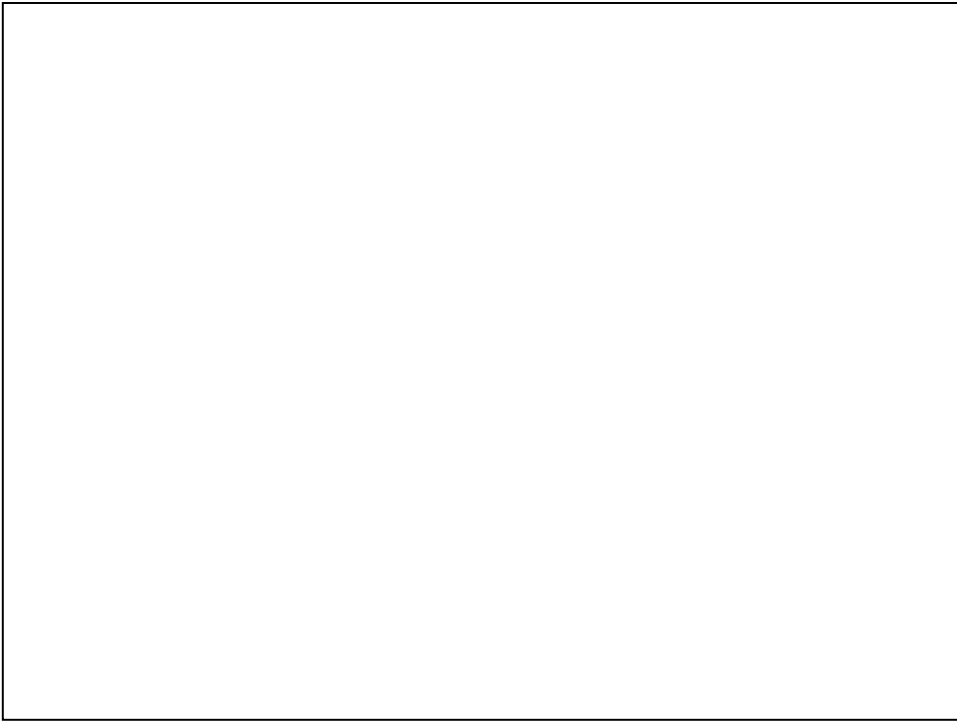




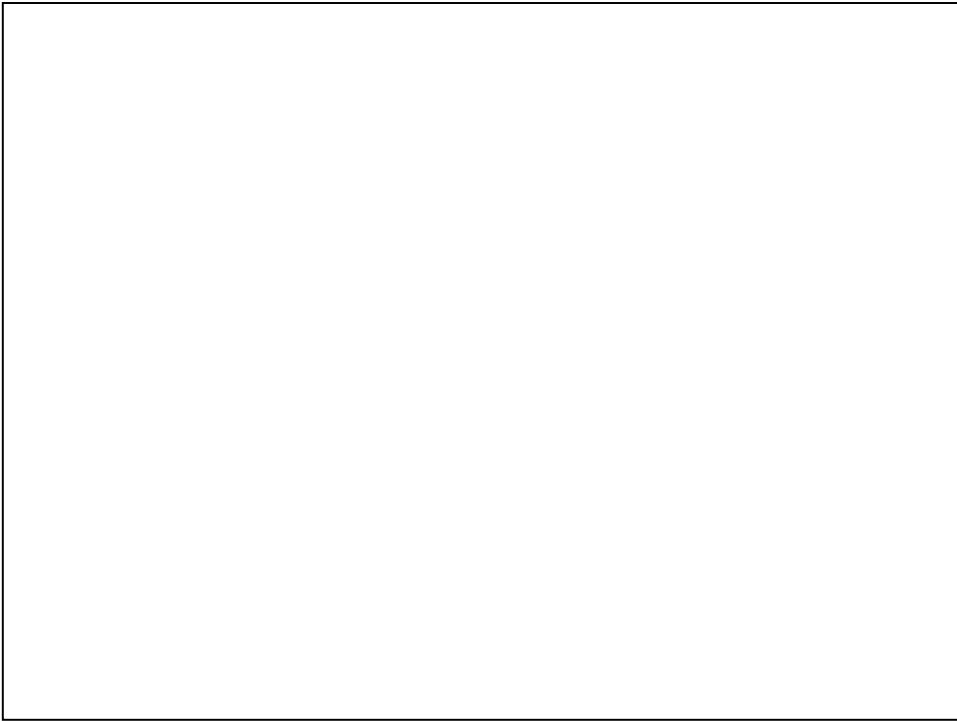


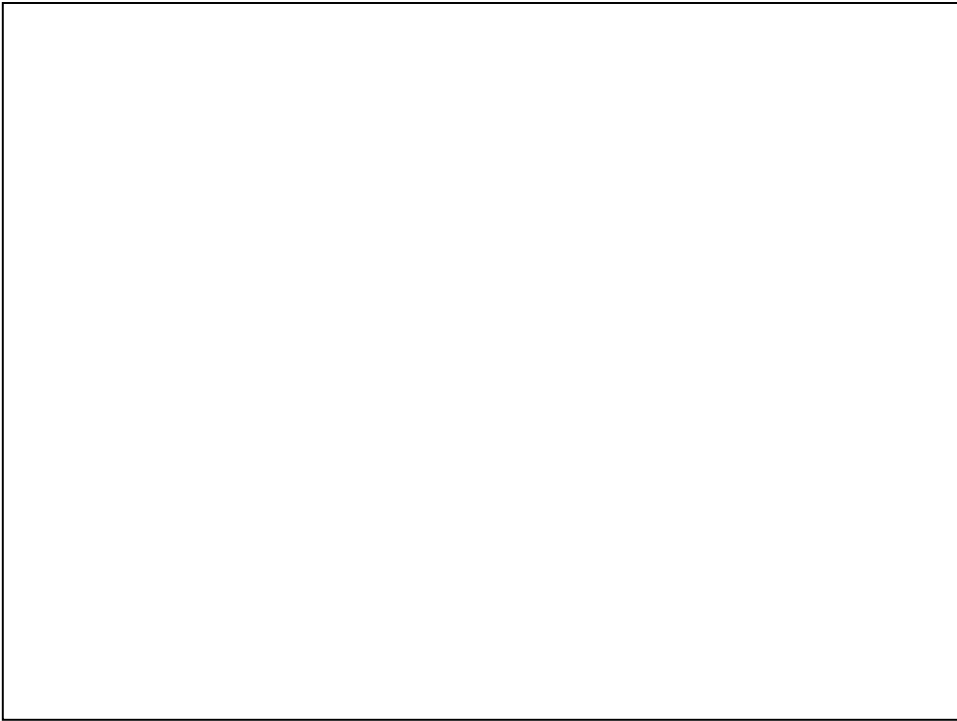














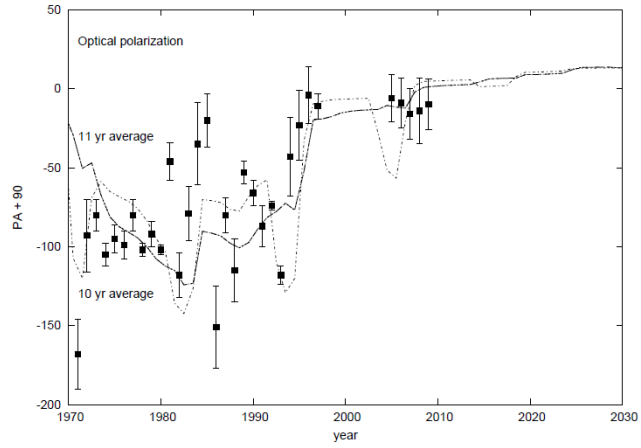
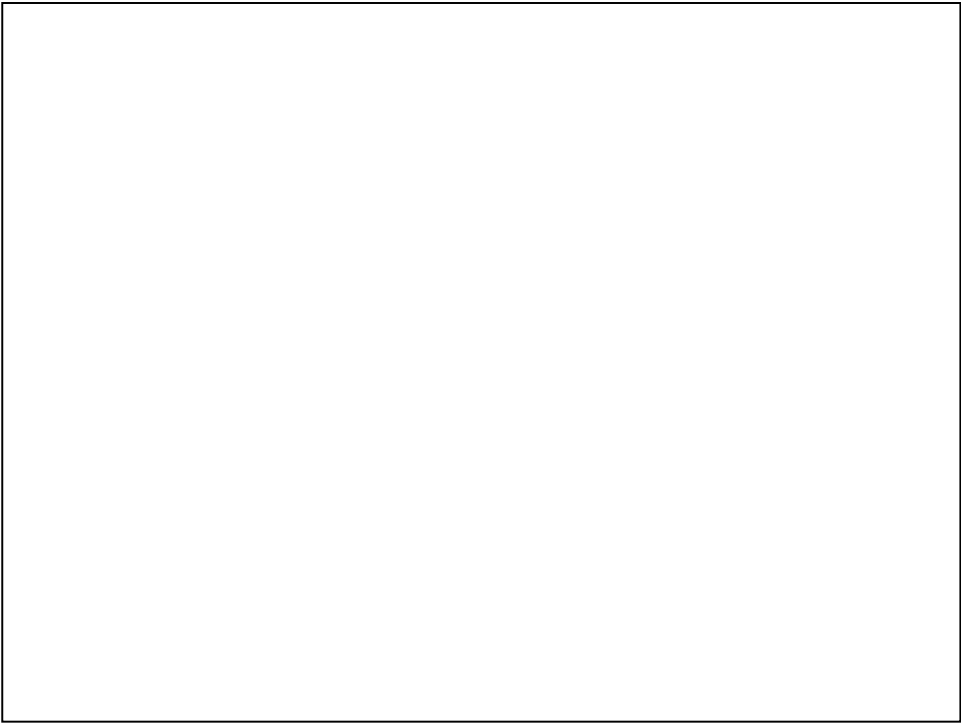
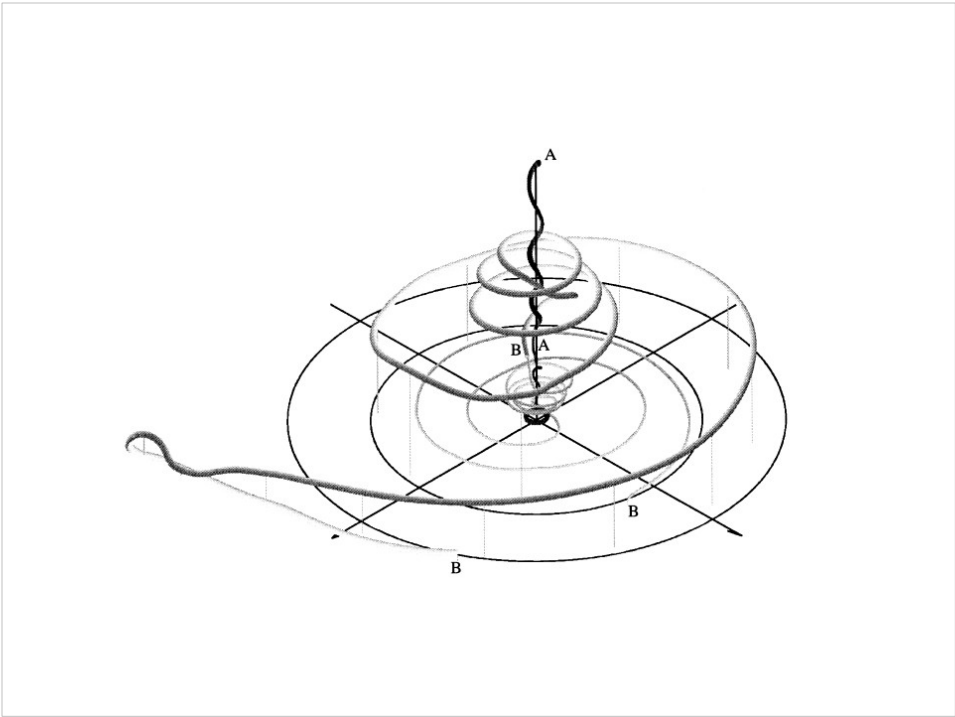
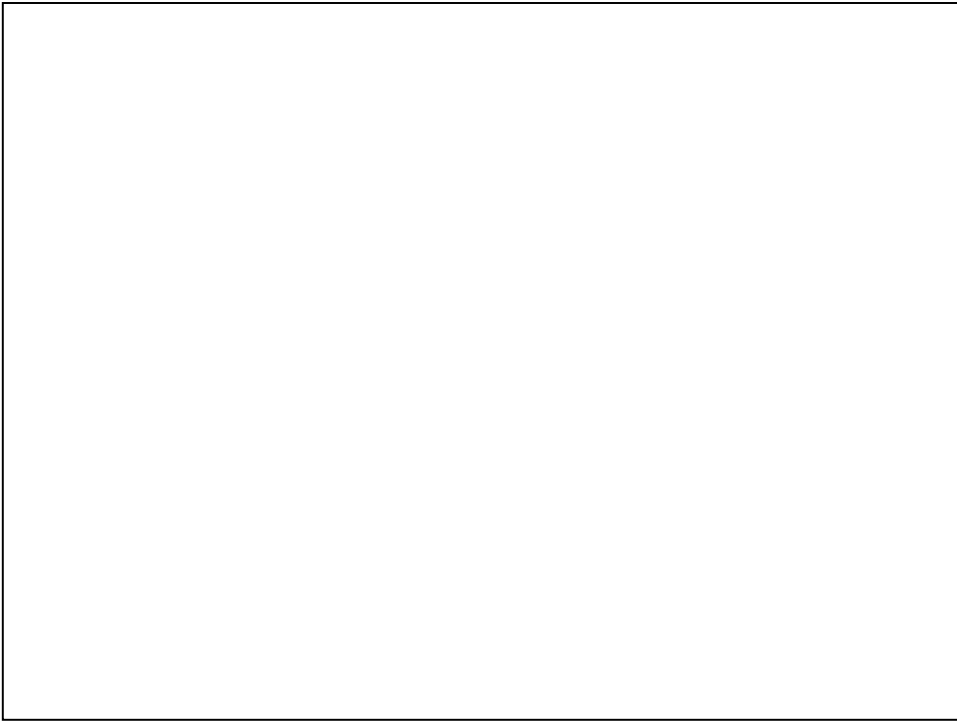


Figure 6. The evolution of the optical polarization angle compared with the model. The curves are for a 10 yr (dash-dot line) and for an 11 yr (dashed line) average. Observations refer to the values in Table 1 minus 180° . The theoretical lines are shifted by adding 90° since the electric vector does not project parallel to the jet axis.















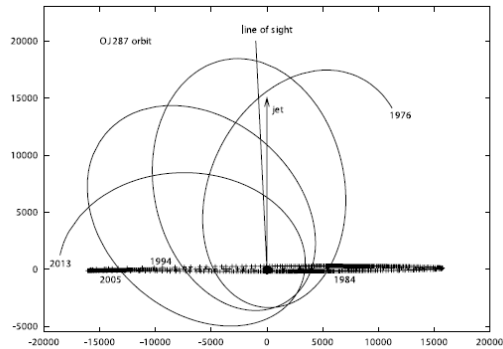
Next: 2015 - 2022

campaign

2015: accurate spin value

2019: test of no-hair -theorem

2022: confirmation of precession



PN acceleration

$$\ddot{\mathbf{x}} \equiv \frac{d^2 \mathbf{x}}{dt^2} = \ddot{\mathbf{x}}_0 + \ddot{\mathbf{x}}_{1\text{PN}} + \ddot{\mathbf{x}}_{\text{SO}} + \ddot{\mathbf{x}}_Q$$
$$+ \ddot{\mathbf{x}}_{2\text{PN}} + \ddot{\mathbf{x}}_{2.5\text{PN}},$$
$$\ddot{\mathbf{x}}_0 = -\frac{Gm}{r^3} \mathbf{x}$$

$$\ddot{\mathbf{x}}_{1\text{PN}} = -\frac{Gm}{c^2 r^2} \left\{ \mathbf{n} \left[-2(2+\eta) \frac{Gm}{r} + (1+3\eta)v^2 - \frac{3}{2}\eta\dot{r}^2 \right] - 2(2-\eta)\dot{r}\mathbf{v} \right\},$$

$$\begin{aligned} \ddot{\mathbf{x}}_{2\text{PN}} = & -\frac{Gm}{c^4 r^2} \left\{ \mathbf{n} \left[\frac{3}{4}(12+29\eta) \left(\frac{Gm}{r} \right)^2 \right. \right. \\ & + \eta(3-4\eta)v^4 + \frac{15}{8}\eta(1-3\eta)\dot{r}^4 \\ & - \frac{3}{2}\eta(3-4\eta)v^2\dot{r}^2 - \frac{1}{2}\eta(13-4\eta) \left(\frac{Gm}{r} \right) v^2 \\ & \left. \left. - (2+25\eta+2\eta^2) \left(\frac{Gm}{r} \right) \dot{r}^2 \right] \right. \\ & - \frac{1}{2}\dot{r}\mathbf{v} \left[\eta(15+4\eta)v^2 - (4+41\eta+8\eta^2) \left(\frac{Gm}{r} \right) \right. \\ & \left. \left. - 3\eta(3+2\eta)\dot{r}^2 \right] \right\}, \end{aligned}$$

Spin-orbit term

$$\begin{aligned}\ddot{\mathbf{x}}_{\text{SO}} = & \frac{G m}{r^2} \left(\frac{G m}{c^3 r} \right) \left(\frac{1 + \sqrt{1 - 4\eta}}{4} \right) \\ & \times \chi \left\{ \left[12 [\mathbf{s}_1 \cdot (\mathbf{n} \times \mathbf{v})] \right] \mathbf{n} \right. \\ & + \left[(9 + 3\sqrt{1 - 4\eta}) \dot{r} \right] (\mathbf{n} \times \mathbf{s}_1) \\ & \left. - \left[7 + \sqrt{1 - 4\eta} \right] (\mathbf{v} \times \mathbf{s}_1) \right\},\end{aligned}$$

$$\ddot{\mathbf{x}}_{2.5\text{PN}} = \frac{8}{15} \frac{G^2 m^2 \eta}{c^5 r^3} \left\{ \left[9v^2 + 17 \frac{Gm}{r} \right] \dot{r} \mathbf{n} - \left[3v^2 + 9 \frac{Gm}{r} \right] \mathbf{v} \right\},$$

$$\ddot{\mathbf{x}}_Q = q \chi^2 \frac{3G^3 m_1^2 m}{2c^4 r^4} \left\{ \left[5(\mathbf{n} \cdot \mathbf{s}_1)^2 - 1 \right] \mathbf{n} - 2(\mathbf{n} \cdot \mathbf{s}_1) \mathbf{s}_1 \right\}$$

no-hair theorem

quadrupole moment Q

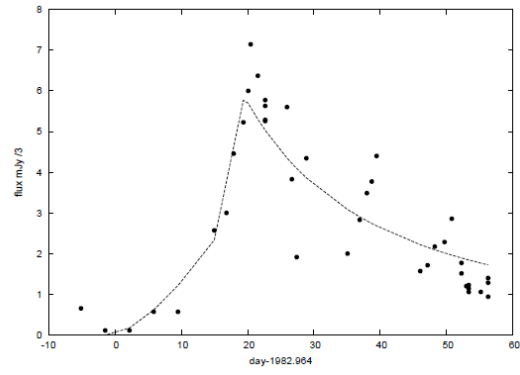
$$Q = -q \frac{S^2}{Mc^2} \quad \mathbf{S}_1 = G m_1^2 \chi \mathbf{s}_1 / c$$

unit vector \mathbf{s}_1

For black holes $q = 1$

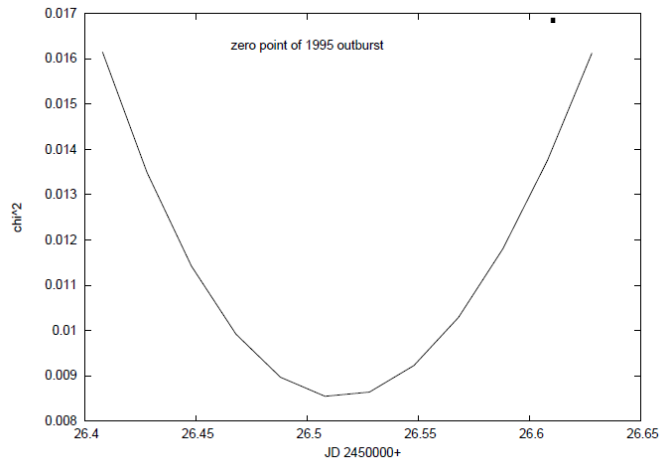
for neutron stars $q > 2$

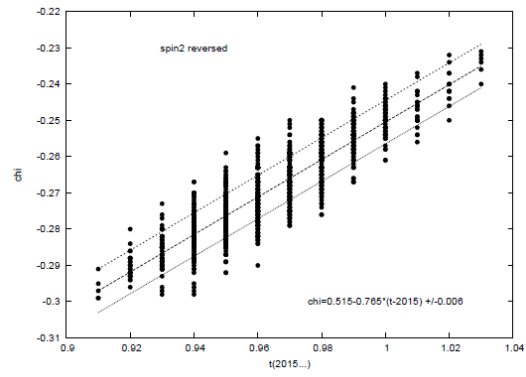


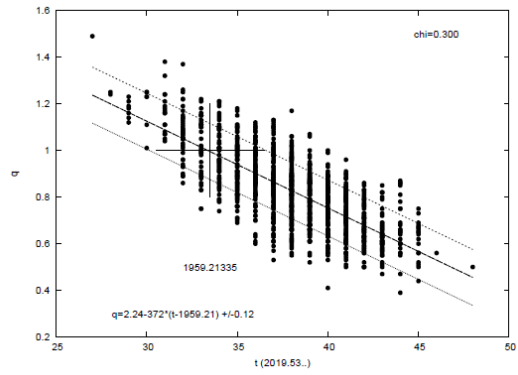


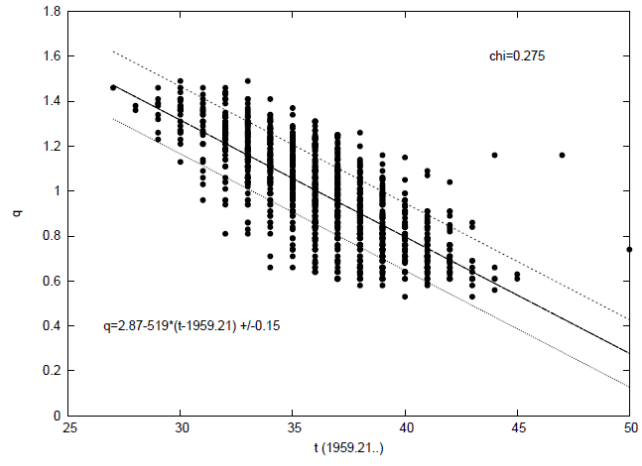


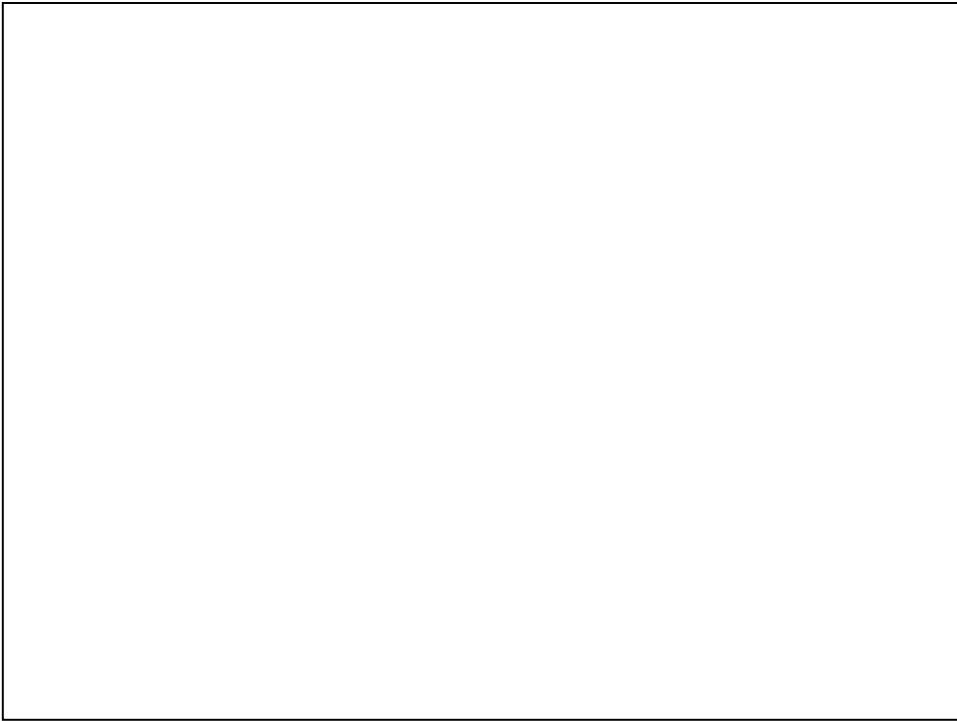
1995.8426

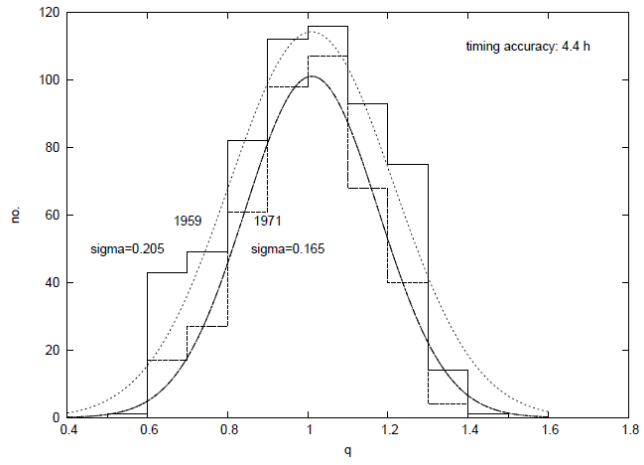


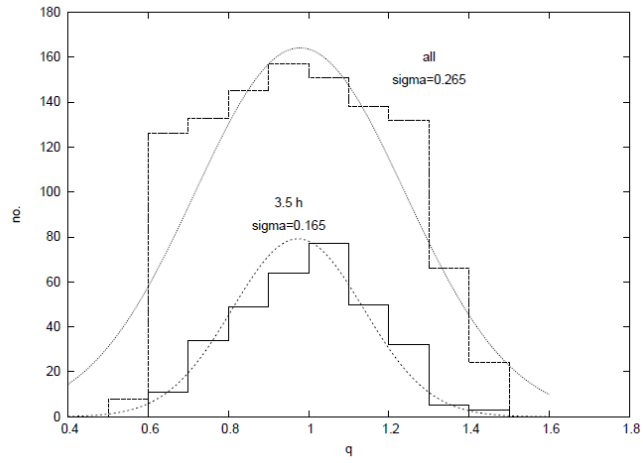


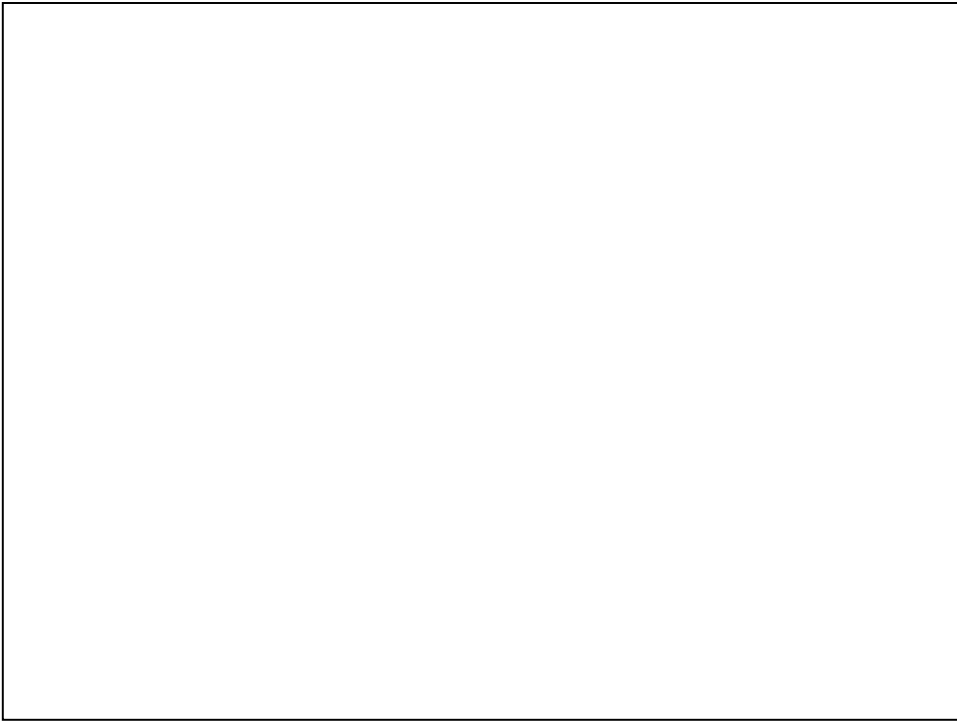


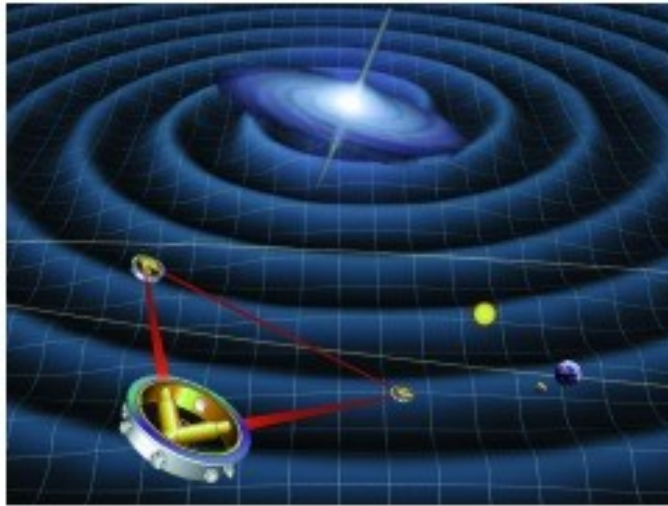












Artist's impression of the LISA spacecraft
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